SPOTTED WING DROSOPHILA MANAGEMENT
IN MICHIGAN BLUEBERRY:
A DYNAMIC STRUCTURAL MODEL

A Thesis

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ABSTRACT

For my Masters thesis, I analyze the management of Spotted Wing Drosophila (SWD) in Michigan blueberry using a dynamic structural econometric model. The Spotted Wing Drosophila (SWD) is a vinegar fly of East Asian origin that can cause damage to many fruit crops. I develop a dynamic structural model to study the SWD management decisions of growers of Michigan highbush blueberry regarding fly and larva monitoring and insecticide application. I apply my dynamic structural econometric model to a detailed data set I have collected and constructed of daily decisions of blueberry growers in Michigan.

BIOGRAPHICAL SKETCH

Shuo Yu was born and raised in Beijing, China. She spent most of her childhood with her grandparents in the rural area of Beijing, where she developed an innate affection for agricultural economics. In college, she pursued a double major in International Trade and Economics and in Accounting at the University of International Business and Economics in Beijing. As an undergraduate research assistant, she wrote a paper on the "Effect of Outward Foreign Direct Investment (FDI) from China on the Rate of Technical Progress of Host Countries", for a project funded by China's Ministry of Commerce. She won a University-level Outstanding Project Award for her project on "Prediction & Analysis of Alibaba Online Sales during Shopping Festival"; and she finished seventh in the Economics and Trade Case Analysis Team Competition. After graduating with a first place ranking out of the 175 students in her undergraduate program in 2017, Shuo came to Cornell University to pursue her M.S. degree in Applied Economics and Management with a major concentration in Food and Agricultural Economics and a minor concentration in Environmental, Energy and Resource Economics. Shuo is interested in applying frontier methodologies to analyze fundamental issues in agricultural and resource economics, and to address sustainable agriculture issues in emerging developing countries.

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Secondly but as importantly, I wish to acknowledge all the informative and fantastic advice provided by Professor C.-Y. Cynthia Lin Lawell, the other member of my committee. Had it not been for her help and effort, I could not have finished this thesis. Taking her Ph.D. class as a Masters student in Spring Semester 2018 helped me to learn about the foundations of structural models and to crystalize the whole method used in my thesis. I am particularly grateful for her support in my life not only as my professor but also as my spiritual mentor.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

Abbreviation (Alphabetical)	**Explanation**
AM	Adaptive Management
BSSM	Bayesian State-Space Modeling
DP	Dynamic Programming
EM Algorithm	Expectation Maximization Algorithm
MI	Michigan State
MLE	Maximum Likelihood Estimation
IPM	Integrated Pest Management
POMDP	Partially Observed Markov Decision Process
SWD	Spotted Wing Drosophila

CHAPTER 1

INTRODUCTION

The Spotted Wing Drosophila (SWD) is a vinegar fly of East Asian origin that can cause damage to many fruit crops. While most Drosophila species are considered harmless or nuisance pests because they are only attracted to spoiled and overripe fruit, SWD exhibits a strong preference for ripe or ripening fruit that has market value (Asplen et al., 2015; Cini et al., 2012).

Given the zero tolerance for larvae in fruit, it is important that growers and processors bring multiple approaches to bear on this pest, to increase the likelihood that fruit are free of contamination. This currently involves combining cultural controls including fruit cooling, fly and larva monitoring, with conventional chemical control. In combination these can help growers and processors meet the market demands (Isaacs et al., 2015).

Incorporating fly and larva monitoring with chemical control is one of the most recommended method for Michigan blueberry growers currently. However, the existing literatures only recommend the starting point and time interval between sprays vaguely.

To fill this gap of the literature, we develop and estimate a dynamic structural model to study the optimal integrated pest management (IPM) timing of SWD management in Michigan highbush blueberry in a single growing season. In particular, we would like to find out when and which type of insecticide to apply conditional on the SWD larva and adult fly monitoring states, aiming to maximize the entire stream of present discounted payoffs in a finite horizon setup. To demonstrate the best timing decision of spraying, we also include other decisions made by farmers that will interact with insecticide application in our model, which are the timing decision of monitoring and harvest. One nuance that can be checked by this model is whether the farmers make decisions in consideration of insecticide resistance and sustainability in the long run.

Our research questions include the following. In the context of SWD management, what is the best timing strategy to apply insecticide and which insecticide to use conditional on the monitoring information? Do growers worry about the potential for developing insecticide resistance?

We will use the structural model to determine whether the farmers are making dynamically optimal decisions or are discounting the future too much by changing different discount factor values and testing for the predictability of the model. We will use parameters estimated from our structural model to conduct counterfactual analysis.

In an extension to our dynamic structural model, we allow for unobserved heterogeneity, which enables us to estimate the distribution of unobserved susceptibility as well as the effects of varietal susceptibility on payoffs.

CHAPTER 2

LITERATURE REVIEW

2.1.SWD Management

Since the detection of the invasive SWD in 2008, quite a few biological models came out regarding its biology characteristics and population development (for instance, Hamby, et al., 2016).

The threat from SWD on blueberries is mainly caused by larval feeding, resulting in the degradation of fruits, since SWD lay its eggs inside ripening fruits, puncturing the fruit's skin with its unique saw-like ovipositor. In addition, the puncturing of the fruit skin also provides a gateway for secondary infections with bacteria and fungi pathogens or additional pests (Atallah et al., 2015; Haye et al., 2016).

Although integrated IPM program of SWD are being developed around the world, including chemical, cultural, and biological control, current SWD management strategies mainly consist of preventive broad-spectrum insecticide sprays (Haye et al., 2016; Van Timmeren and Isaacs, 2013).

In recent MSU trials, several different kinds of registered insecticides have shown excellent control against SWD which fall into four categories, organophosphate, pyrethroid, diamide and spinosyn insecticides. Different insecticides present divergence in efficiency and application costs consisting of labor cost and material cost. And rotation in insecticides is useful in resistance management. Monitoring of SWD larva and adult flies are becoming prevailing to be incorporated with insecticide application in order to minimize the damage from both fruit infection and overuse of insecticide that may increase insecticide resistance and harm workers and consumers' health. Currently, monitoring of SWD activity is based on sampling fruit for SWD larva and trapping methods for SWD adult flies. (Isaacs et al., 2015)

To capture the population growth information from the partially observed states through the sampling and monitoring, there are several approaches developed, including Partially Observed Markov Decision Process (POMDP), adaptive management (AM) and Bayesian state-space modeling (BSSM). BSSM offers a framework to simultaneously address population uncertainty and partial observability and has been extensively used in statistical ecology. (Fan, et. al., 2016) One of the nuances of this paper is to incorporate BSSM in transition density estimation.

2.2.Dynamic Structural Model

The dynamic structural model used in this paper will apply the nested fixed-point maximum likelihood estimation approach, which is first developed by Rust (1987). The original paper applied this method to a simple regenerative optimal stopping model of bus engine replacement and found the solution to a stochastic dynamic programming problem that formalizes the trade-off between the conflicting objectives of minimizing maintenance costs versus minimizing unexpected engine failures. This dynamic structural model was applied to many different contexts since then, including water management (Timmins, 2002), land use in agriculture (Scott, 2013), agricultural productivity (Carroll et al., 2019a), wind turbine shutdowns and upgrades (Cook and Lin Lawell, 2019), crop disease control (Carroll et al., 2019c), pesticide spraying decisions (Sambucci et al., 2019), and supply chain externalities (Carroll et al., 2019b).

Arcidiacono and Miller (2011) innovates upon the dynamic structural econometric model in Rust (1987) by allowing for unobserved heterogeneity. Sambucci et al. (2019) develops and applies a dynamic structural econometric model with unobserved heterogeneity to analyze pesticide spraying decisions of grape growers.

CHAPTER 3

EMPERICAL APPROACH

3.1.Background

3.1.1. SWD IPM Program in Michigan Blueberry

Michigan blueberry IPM program against SWD mainly consists of monitoring, spraying and other cultural methods to remove leftovers.

Serving as an alarm of the start of fly activity, SWD adult fly monitoring are always carried out using traps and lures, from after fruit set until the end of harvest. The traps and lures are available to be purchased from commercial suppliers or homemade easily at very low price, less than $10 per trap with lure. Traps for SWD should be hung in a shaded area in the fruit zone, using a wire attached to the top of the trap, with a minimum of one trap every 5-10 acres. They should be checked for SWD flies at least once a week. (Isaacs et al., 2015)

To monitor whether the fruit are infested and how serious is the infestationthe infestation is, the growers are recommended to do fruit sampling and salt solution testing. After lightly crushed berries immersed in the salt solution for at least 30 minutes, the larva will float in the liquid making them easier to see. (Isaacs et al., 2015)

Once fruit are ripening and SWD flies are present, registered insecticide application will be needed to minimize the risk of infestation until the end of the harvest season. With these methods enabling partial observation of the pest population, the growers are exposed to more information assisting precise chemical control methods. Growers can also consider post-harvest controls including temperature treatment and soft-sorting machinery.

For both adult and larva monitoring, to escalate valid and insightful information, the growers or the extension researchers may sample both inside and at the edge of the plots. The only difference of monitoring larva inside the fields comparing to that at the edge of the fields is that the data collectors need to go inside the field (just a few tens of feet away) to obtain the sampling fruits. All the other appliances and procedures are all the same, such as the fruit dunk flotation method or boil test. Hence, the costs for both operations are regarded the same in our paper.

We may expect different pest densities between these locations due to the following reasons:

(1) According to Rufus Isaacs et al. (2015), they have also observed higher catches in traps adjacent to fields where they remain wet longer, or adjacent to creeks. Because of the worse drainage and ventilation conditions inside the fields comparing to at the edge of the fields, we may expect higher observations inside the fields.

(2) According to Rufus Isaacs et al. (2015) and my interview with Bob Carini (Carini Farms), the neighboring wild host plants can harbor SWD such as wild grape, pokeberry, honeysuckle, nightshade, dogwood, spicebush, autumn olive, raspberry, blackberry, etc. near crop fields; and if the neighboring farms have not applied enough insecticides, the over-ripened fruit are not treated correctly, or the infestation is not controlled there, they can also become sources of infestation risk. In these cases, we may expect higher observations at the edge of the fields.

However, in our 2018 Michigan blueberry data, little difference between the samples inside the fields and those at the edge is detected, so we could consider them subsamples of the same fields which elevates the accuracy of the data.

In the current situation of Michigan blueberry production, most farmers may take up the adult fly monitoring using traps and lures, but the fruit sampling methods for larva monitoring are basically done by researchers at MSU. The adult fly monitoring may give the farmers an early alarm. After observing the presence of SWD, most farmers will spray either according to the scouting and reports released weekly by MSU or just following the calendar. The information from MSU are well penetrated with 70%-80% farmers well covered by MSU extension institute education events in the area.

Basically, they will use the effective and cheaper insecticides as much as they need and as little as they can. There is a strong economic incentive for the growers both to spray since the infested blueberries will be totally rejected by the market and lead to huge lost, yet they do not want to over spray. Each spray will cost them about $100 per acre including insecticide cost and application costs, which can may be a small portion comparing to the revenue but will kill the profit significantly.

The farmers do control for resistance by changing insecticide chemical category, but there is no specific rotation order that is a standard one or recommended one.

3.1.2. The Highbush Blueberry Market/Pricing Structure in Michigan

Michigan grows many different blueberry varieties, spreading the harvest season over several months (As shown in the figure below). Not all the fruit on a bush ripens at once and each variety can be harvested for 3 to 4 times per season, which lasts for 2 to 3 weeks. The first harvest often takes place when there are 25 percent of the berries are ripe and is typically by hand. These early harvest blueberries are most likely sold on fresh market where they will gain a good price. And the other harvests later are probably done by machines and will probably go to the processed market. To have fruit for several months, growers usually plant multiple cultivars with staggered and overlapping harvests. As shown in the figure below, highbush blueberries can endure lower temperature and there is later-season varieties whose harvest season can be postponed to mid-September. (Mark Longstroth, 2016)

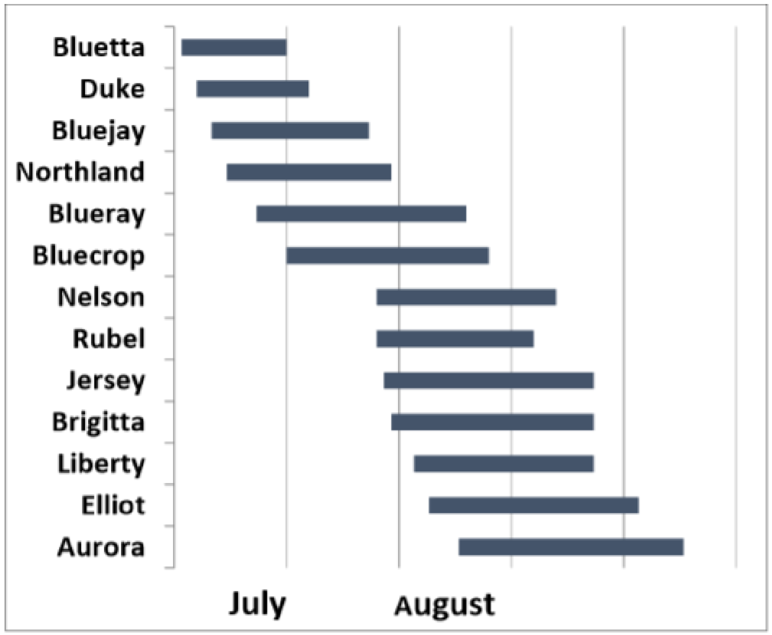

Figure 1 Blueberry Harvest Dates at South Haven

Source: Longstroth, Mark. "The Michigan Blueberry Industry in 2018." Michigan State University Extension (2018): 5.

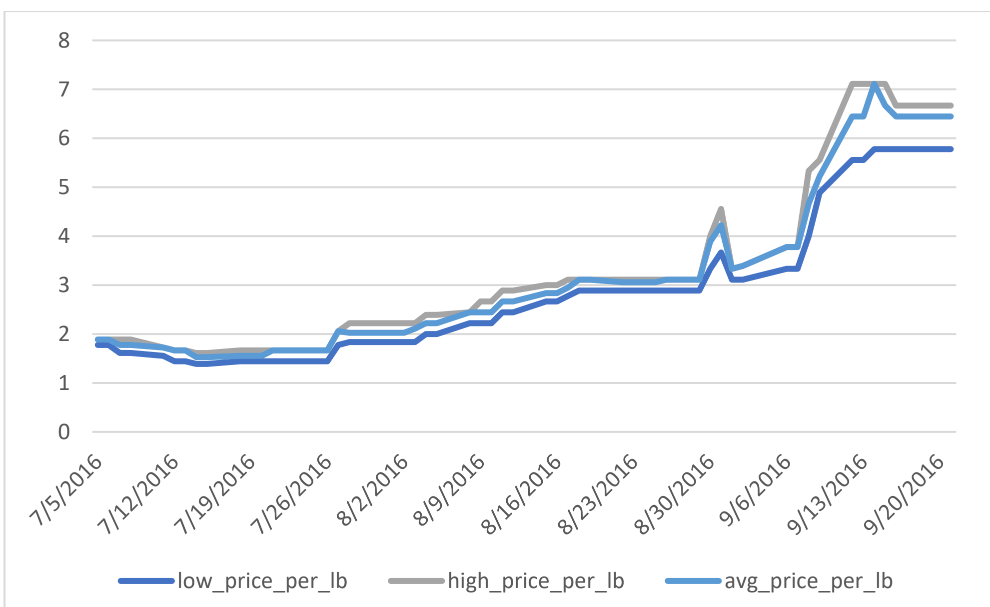

Figure 2 2016 Unit Blueberry Price Originated in Michigan

Data Source: USDA Fruit and Vegetable Market News Portal

In Michigan's southern peninsula, first SWD fly activity is typically in mid-June to early July and the population builds through the summer as temperatures continue to rise. Highest densities of SWD occur in August and September, so SWD is especially problematic for later-season berry crops, including blackberries, fall raspberries, ever-bearing strawberries, and late-season blueberries.

Therefore, the later season blueberry crop may be exposed to higher risk of SWD infestation while entertaining a higher price on the market at that time. Thus, the farmers may also need to decide which type of blueberries to grow and when to harvest.

However, few growers are willing to replant new cultivars against the infestation of SWD. To initiate, the highbush blueberries are long-lived and can be productive for over 60 years (the oldest fields at south western Michigan is about this age), with being the most productive in their twenties. What's more, the growers have to bare not only the high replanting cost but also the first five to six years' maintenance expenses without any receipts, if new cultivars are planted, and thus the costs for replanting are almost prohibitive. Even if the grower decides to change to earlier or later varieties, they will only change a very small portion of their blueberry crops. What's more, with more and more blueberries imported from Peru and Mexican, the price of later season cultivar is at pressure. Therefore, in this paper we simplify the model by ignoring the replanting decision.

In addition, price per pound of a particular blueberry cultivar is unavailable, since blueberries in the United States are not sold with type and origins labeled. On the one hand, the blueberries of different cultivars look very similar and only differs in taste. On the other hand, the farmers are not willing to pamper their consumers to wait until their favorite cultivars come to the market.

3.1.3. Different Blueberry Cultivars

In the main producing areas of Michigan blueberry, each grower divides their farms into several plots for different cultivars with distinguished harvesting intervals, so as to ensure consecutive market delivery of fresh blueberries from early May to late September (Figure 1).

Therefore, there may exist some interactions between plots due to the distinguished harvest season. Since the SWD may lay eggs regardless of the variety of blueberries, no matter what the cultivar is on the plots, once a plot is infested, the risk of its neighboring plots will increase. Keeping that in mind, if the early cultivars, say Blue Crop, on the farm are infested, the grower may apply more insecticides to the late cultivars, like Elliott. Blueberries are not threatened by infection before the fruits are colored, but once the early varieties are harvested, the pests will transfer to the adjacent late varieties. It can be regarded as an early alarm of the risk for the late varieties. (According to the interview with Bob Carini (Carini Farms).)

3.2.Intuition and Tradeoffs

As shown above, there are three types of decision investigated in this paper, including monitoring decision, insecticide application decision and harvest decision. For each time period (each day), the farmer's action will be a combination of all these three decisions.

For each time period, if the farmer decides to monitor the adult flies or the larva instead of waiting to monitor in a later time period, she will incur a monitoring cost consisting of material cost and labor cost but will lose the opportunity to gain a better information about the pest population. A partially observed number of adult flies' population will be an effective early alarm of SWD activity, and the fruit sampling application crystalizes the contamination degree of the planting outcomes. Both information will facilitate the pest management and reduce the risk of an entire rejection of the fruits by markets.

As for the benefits and costs analysis on apply insecticide application, on the one hand, the farmer will bear the material cost, labor cost and sprayer cost, if she decides to spray, which are specific to the type of insecticide she choose to apply. On the other hand, the insecticide application could help to forestall the detriment caused by the growing population of SWD, which will bring serious yield losses during the harvesting stage. In contrary, if the farmer chooses not to apply any insecticide at that time period, there will be no cost incurred on that day, but she will be at the risk of higher damage due to SWD infestation at the end of the growing season. Therefore, the main tradeoff governs a farmer's decision to spray a particular insecticide against to wait is between the costs of insecticide application incurred at this time period and the risk of higher blueberry damage due to SWD infestation at the end of growing season, which will be shown by the discounted revenue loss in the continuation value.

Another tradeoff considered when choosing a particular insecticide over another is between the costs incurred for the specific insecticide applied and the risk of higher damage due to insecticide resistance, since we assume that if the grower sprays the same cheaper insecticide consecutively, the SWD population may become more resistant to the particular insecticide, and thus results in lower yield.

Lastly, since on the time period that the grower chooses to harvest, she receives the revenue determined by the yield and market price, and the uncertainty of damage due to SWD is realized, the harvest timing is also a critical decision to make by the farmer. This choice is governed by two opposing forces. If the farmers decide to harvest earlier, they will confront lower risk of contamination since the population of SWD haven't reached its peak at that time and trim off the costs of monitoring and spraying due to fewer control measures demanded against SWD, while they will simultaneously let go the premium profits boosted by price advantage in the later season.

All these decisions have an invertible nature similar to investment, with sunk cost incurring at the decision made. What's more, once the decision is made, the grower cannot recover it all should she change her mind. Since the state variables are evolving stochastically over time, there is uncertainty over the future rewards as detailed above. Thus, there will be leeway over timing of monitoring, spraying, harvest, or some combination of these decisions, for the growers can postpone their actions to get better information about the future. Therefore, the structural dynamic model will help us to capture all these characteristics of the decision-making process and allow for the invisible opportunity cost of waiting to take the actions later.

3.3.Data

The main data concerning highbush blueberry IPM program in Michigan State used in this paper consisted of two part, one from 2016 2-sample farmer survey data, and the other from 2018 Michigan State University Extension collected data. To form the final panel data used for our structural model construction and analysis, we also collected data concerning pesticide characteristics and cost estimations of all sorts, including machinery, labor and materials, from both open source websites, and private visits and interviews with the farmers, processors, extension educators and experts from Entomology department.

The Michigan 2016 and 2018 data we used, and their sources are listed as in Tables 1 and 2. In accordance with the requirement of the structural models, we manipulated them into daily panel data with the time horizon in 2016 being from June 1^{st} to August 31^{st}, which is 92 days, and that in 2018 being from July 1^{st} to September

15th, which is 76 days. In 2016 data, we have two samples in the survey data, which consists of 184 observations. And for the 2018 data, we have 6 growers with 3 different

Table 1 2016 Michigan Highbush Blueberry Data Descriptions and Sources

Variable	Description	Measurement	Source
Basic information	Bearing season, acreage	Daily based	Philip Fanning and Rufus Isaacs. 2016. MSU Grower Survey.
Crop stages	Full bloom, early green fruit, late green fruit, fruit coloring, harvest	Daily based	Philip Fanning and Rufus Isaacs. 2016. MSU Grower Survey.
Insecticide application	Date applied, insecticide brand, total amount, unit price($/oz), efficiency against SWD, time spent (labor hours)	Daily based	Philip Fanning and Rufus Isaacs. 2016. MSU Grower Survey.
SWD monitoring	Date, costs, adult SWD captured, larva sampling size and number found	Daily based	Philip Fanning and Rufus Isaacs. 2016. MSU Grower Survey.
Insecticide information	Insecticide efficiency, class, PHI days, minimum days between sprays, days of activity, and etc.	By year	Rufus Isaacs, John Wise, Carlos Garcia-Salazar, and Mark Longstroth. 2015.06. SWD Management Recommendations for Michigan Blueberry.
Costs	Monitoring cost, spray cost.	By year	Philip Fanning and Rufus Isaacs. 2016. MSU Grower Survey.
			Mark Longstroth. 2018. Cost Analysis of Blueberry Potential Profit.
			Bureau of Labor Statistics, Department of Labor. 2017. Occupational Employment Statistics (OES) Survey.

varieties each amounting to 18 different grower-variety combinations. Since the monitoring and spraying decisions of the same grower across different varieties are independent, we can model the decision-making of each grower-variety as separate decision-making problems. Thus, we have 18 samples in 2018 data, which add up to 1,386 observations.

Table 2 2018 Michigan Highbush Blueberry Data Descriptions and Sources

Variable	Description	Measurement	Source
Basic information	Crop, bearing season, acreage, weather	Daily based	Philip Fanning and Rufus Isaacs. 2019.02. 2018 Spray Records.
Insecticide application	Date applied, insecticide brand, total amount, unit price($/oz), efficiency against SWD, time spent (labor hours)	Daily based	Philip Fanning and Rufus Isaacs. 2019.02. 2018 Spray Records.
SWD monitoring	Date, costs, adult SWD captured, larva sampling size and number found	Daily based	Philip Fanning and Rufus Isaacs. 2019.02. 2018 SWD Fruit Assessment. Philip Fanning and Rufus Isaacs. 2019.02. 2018 SWD Study Trap Data.
Insecticide information	Insecticide efficiency, class, PHI days, minimum days between sprays, days of activity, etc.	By year	Rufus Isaacs, John Wise, Carlos Garcia-Salazar, and Mark Longstroth. 2015.06. SWD Management Recommendations for Michigan Blueberry.
Costs	Monitoring cost, spray cost.	By year	Philip Fanning and Rufus Isaacs. 2019.02. 2018 Spray Records.
			Mark Longstroth. 2018. Cost Analysis of Blueberry Potential Profit.
			Occupational Employment Statistics (OES) Survey. Bureau of Labor Statistics, Department of Labor

The following are the descriptive statistics of main variables (Tables 3 and 4). I will further detail the discretized variables *fly_f_discrete*, *fly_m_discrete* and *lar_discrete_total* later in the variable section.

Table 3 Descriptive Statistics of Main Variables for 2016 Data

	count	mean	std	min	Percentiles			max
					25%	50%	75%	
choice_ins	184	0.15	0.48	0	0	0	0	2
choice_mon_adu	184	0.10	0.30	0	0	0	0	1
choice_mon_lar	184	0.07	0.26	0	0	0	0	1
int_ins	184	37.22	44.80	1	3	7	99	99
int_mon_adu	184	31.01	42.83	1	3	6	99	99
int_mon_lar	184	49.08	47.43	1	4	8	99	99
last	184	-32.89	47.84	-99	-99	1	2	2
fly_females	184	-23.86	47.63	-99	-99	2	6	25
fly_f_discrete	184	-27.32	45.11	-99	-99	1	1	2
fly_males	184	-24.91	46.96	-99	-99	1	4	25
fly_m_discrete	184	-27.40	45.07	-99	-99	1	1	2
lar_rate	184	-46.76	49.61	-99	-99	0	0	1
lar_discrete	184	-46.76	49.61	-99.00	-99.00	0.00	0.00	1.00

Table 4 Descriptive Statistics of Main Variables for 2018 Data

Variables	count	mean	std	min	Percentiles			max
					25%	50%	75%	
choice_ins	1386	0.04	0.26	0	0	0	0	2
choice_mon_adu	1386	0.05	0.21	0	0	0	0	1
choice_mon_lar	1386	0.04	0.20	0	0	0	0	1
int_ins	1386	64.03	34.26	1	76	76	76	99
int_mon_adu	1386	57.06	40.78	1	6	76	99	99
int_mon_lar	1386	59.87	41.85	1	6	76	99	99
last	1386	-36.62	48.73	-99	-99	1	2	2
fly_females	1386	-28.67	45.53	-99	-99	0	0	43
fly_f_discrete	1386	-28.33	45.66	-99	-99	1	1	2
fly_males	1386	-28.73	45.49	-99	-99	0	0	42
fly_m_discrete	1386	-28.36	45.65	-99	-99	1	1	2
lar_rate	1386	-38.13	48.21	-99	-99	0	0	0
lar_discrete	1386	-38.14	48.20	-99.00	-99.00	0.00	0.00	0.00

3.4.Empirical Strategies

In this model, the farmer makes discrete decisions on whether to apply insecticide and which category of insecticide to apply each day, based on state variables either observable or unobservable to econometricians. This decision-making process has the irreversible nature and involves uncertainty over the future rewards, since the application can affect the possibility of SWD infection of blueberries, and thus will significantly affect the revenue of the plots. Therefore, the decision can be regarded as a dynamic investment decision-making process that is very similar to the one described in Sambucci et al. (2019).

3.4.1. Assumptions

a) The blueberry crop is under the risk of infection from the beginning of fruit coloring stage to the end of harvesting season since female SWD can only lay eggs in ripening or ripened fruits, so the risk of infection never appears before the full bloom stage, we make time period 0 be the time when the blueberries enter the full bloom stage.

There is no effect between different growing seasons, so each grower-year combination will be treated as a different decision-making process.

b) There is no spatial externality between growers in the same neighborhood, including technology spillovers, SWD population migration and etc.

Assumption b) and c) ensures that each grower-year combination will be treated independently as a dynamic individual agent investment decision-making process instead of a dynamic game between growers.[1]

[1] Since for each year-grower combination, the decision-making process is an isolated dynamic optimization system. We omit the index of grower, i, from here on for succinctness.

Starting from the time period t_0 when the blueberries are planted until the last time period T when the harvest season starts. The grower $i\left(i \in \{1,...,I\}\right)$ chooses a sequence of combinations of four discrete decisions $\left\{\gamma_0, \gamma_1 \ldots, \gamma_T\right\}$, where $t \in \{0,...,T\}$ is the index of time period which is the number of days starting from the initiation of full bloom stage in each growing season, to maximize the discounted present value of the entire stream of per-period payoff

$$\max_{\{\gamma_t \in \Gamma_t\}_{t=0}^{T}} E\left\{\sum_{t=0}^{T+1} \beta^t \pi\left(\gamma_t, \overrightarrow{x_t}, \overrightarrow{z_t}, v, \varepsilon_t; \theta\right)\right\}$$

where $\overrightarrow{x_t}$ and $\overrightarrow{z_t}$ are vectors of observable exogenous and endogenous state variables that influence the probability of SWD infection level respectively; v is an unobserved time-invariant state variable measuring susceptibility to SWD specific to each farm; ε_t is a vector of random shocks $\varepsilon_t\left(\gamma_t\right)$ to per-period utility, one for each possible action at in the action set, that is observed by the grower, but not by the econometrician; and θ is a vector of parameters to be estimated.

3.4.2. Choice Variables

For each time period t, each grower i will decide whether or not to monitor. Demanding different methods and serving for distinguished purposes, monitoring for larva and adult flies may be taken on different days, so we separate the monitoring decision into two different choice variables. We denote the decision of monitoring for SWD adult flies by

$$a_t \in A, \text{ where } A = \{0,1\},$$

and that of monitoring for SWD larva by

$$b_t \in B, \text{ where } B = \{0,1\}.$$

For each time period t, each grower i will choose one of the three common types of insecticide to spray or choose not to spray and to wait until later. We incorporate this decision with three different values for insecticides application choice variable. Thus, we have

$$c_t \in C, \text{ where } C = \{0,1,2\},$$

In 2016 data, the values are defined as

$0 :=$ not to apply insecticide;
$1 :=$ to apply organophosphate insecticide;
$2 :=$ to apply pyrethroid insecticide.

And in 2018 data, the values are defined as

$0 :=$ not to apply insecticide;
$1 :=$ to apply Mustang Maxx insecticide;
$2 :=$ to apply Brigade 2EC insecticide.

The harvest seasons of highbush blueberries usually last for around three weeks, intriguing often about three harvests for each season with first two by hand and the last one by machine, and the harvest interval is highly relied on the blueberry cultivars. As stated above, growers are reluctant to substituting the exiting crops with other varieties just regarding the harvest season due to the high replanting cost. In sight of this, we mainly capture the harvesting time difference by adding variety dummy variables for fixed effect, instead of adding another choice variable for the harvest action.

The actual discrete action is a combination (tuples) of all the action variables above. Each grower will choose exactly one of these action tuples each day. We denote it by

$$\gamma_t = (a_t, b_t, c_t), \textit{ where } \gamma_t \in \Gamma = A \times B \times C.$$

3.4.3. Observable State Variables

The observable state variables are divided into exogenous ones, $\overrightarrow{\mathrm{x}_t}$, and endogenous ones, $\overrightarrow{\mathrm{z}_t}$. The exogenous variables are assumed to evolve as finite state first-order Markov processes, with independent identically conditional distribution $F_x(\overrightarrow{x_{t+1}} \mid \overrightarrow{x_t})$. The exogenous variables are as following:

a) Maximum days of activity of the insecticide for each insecticide,

max_active_O, max_active_P in MI 2016 data, and $max\_active\_mustang$, $max\_active\_brigade$ in MI 2018 Data.

b) Cost for each possible monitoring and spraying choices for the whole plot (whether or not the grower actually chooses that action),

$cost\_mon\_adu$, $cost\_mon\_lar$, $cost\_org$, and $cost\_pyr$ in MI 2016 data;

$cost\_mon\_adu$, $cost\_mon\_lar$, $cost\_mustang$, and $cost\_brigade$ in MI 2018 data.

The costs include labor cost, material cost and machinery cost, which are specific to each grower-year combination and corresponding to their valid actions.

The endogenous variables, $\vec{z}_t$, are assumed to evolve as finite state first-order Markov processes, with independent identically conditional distribution $F_z(\vec{z}_{t+1} | \vec{x}_t, \vec{z}_t, g_t; q)$. The endogenous variables $\vec{z}_t$ are shown as below:

a) Interval since last monitoring, $int\_mon\_adu_t$ and $int\_mon\_lar_t$.

These variables transited deterministically,

$$int\_mon\_adu_t = \begin{cases} 99 & \text{, if haven't monitored in the year} \\ int\_mon\_adu_{t-1} + 1, & \text{if } a_{t-1} = 0 \\ 1 & \text{, if } a_{t-1} = 1 \end{cases},$$

$$int\_mon\_lar_t = \begin{cases} 99 & \text{, if haven't monitored in the year} \\ int\_mon\_lar_{t-1} + 1 & \text{, if } b_{t-1} = 0 \\ 1 & \text{, if } b_{t-1} = 1 \end{cases}.$$

Since the monitoring data is weekly based, the number of days since last update of the monitor information may change the grower's estimation of the SWD population size and the expectation of SWD infection in the future.

We code them as "99" if the grower has not yet monitored this season, which is much longer than the maximum days of activity for any possible insecticide, and which represents that the interval since last monitoring is greater or equal to 99 days. In other words, if the grower has not yet monitored this season, we assume any monitoring from any previous season does not apply for this season and therefore we will just choose a large number for $int\_mon\_adu_t$ and $int\_mon\_lar_t$ to represent this.

In the 2018 data, we code any intervals greater than 10 to be 76 which is the length of the time horizon for each grower-variety combination in the 2018 MI data. That is,

$$int\_mon\_adu_t = \begin{cases} 99 & \text{, if haven't monitored in the year} \\ int\_mon\_adu_{t-1}+1, & \text{if } a_{t-1}=0 \text{ and } int\_mon\_adu_{t-1}<10 \\ 76 & \text{, if } a_{t-1}=0 \text{ and } int\_mon\_adu_{t-1}\geq 10 \\ 1 & \text{, if } a_{t-1}=1 \end{cases}$$

$$int\_mon\_lar_t = \begin{cases} 99 & \text{, if haven't monitored in the year} \\ int\_mon\_lar_{t-1}+1 & \text{, if } b_{t-1}=0 \text{ and } int\_mon\_lar_{t-1}<10 \\ 76 & \text{, if } b_{t-1}=0 \text{ and } int\_mon\_lar_{t-1}\geq 10 \\ 1 & \text{, if } b_{t-1}=1 \end{cases}$$

The reason for this manipulation is to not only reduce the dimension of the dimension of the functions but also magnify the different influences of interval length that are greater than the maximum active interval of the insecticides on the utility function, since we believe that the risk will increase if the information is invalid when the crops are no longer effectively protected by the insecticides.

b) The interval since last insecticide application, int_ins_t.

The interval since last insecticide application is an endogenous variable, measured in days and has a deterministic pattern of evolvement:

$$int\_ins_t = \begin{cases} 99 & \text{, if haven't applied insecticide in the year} \\ int\_ins_{t-1}+1 & \text{, if } c_{t-1}=0 \\ 1 & \text{, if } c_{t-1}\geq 1 \end{cases}$$

Similarly, as in a), we code it as "99" if the grower has not yet applied insecticide this season, which is much longer than the maximum days of activity for any possible insecticide, and which represents that the interval since last spraying is greater or equal to 99 days. In other words, if the grower has not yet applied insecticide this season, we assume any insecticide from any previous season has long worn off, and therefore we will just choose a large number for int_ins_t to represent this.

In the 2018 data, we code any intervals greater than 10 to be 76 which is the length of the time horizon for each grower-variety combination in the 2018 MI data. That is,

$$int\_ins_t = \begin{cases} 99 & \text{, if haven't applied insecticide in the year} \\ int\_ins_{t-1}+1 & \text{, if } c_{t-1}=0 \text{ and } int\_ins_{t-1}<10 \\ 76 & \text{, if } c_{t-1}=0 \text{ and } int\_ins_{t-1}\geq 10 \\ 1 & \text{, if } c_{t-1}\geq 1 \end{cases}$$

The reason for this manipulation is to not only reduce the dimension of the dimension of the functions but also magnify the different influences of interval length that are greater than the maximum active interval of the insecticides on the utility function, since we believe that the risk will increase dramatically (not linear relationship) if the crops are no longer effectively protected by the insecticides.

c) Discretized SWD larva found rate in the latest monitor process, $lar\_discrete_t$, discretized female SWD trapped, $fly\_f\_discrete_t$, and discretized male SWD trapped, $fly\_m\_discrete_t$.

The farmer uses saltwater extraction method to monitor SWD larva amount on a weekly base and it is reported as a rate (amount of larva found/sample size). The farmer uses traps to monitor SWD fly amount on a weekly base and they are reported as number of flies trapped per trap. They may affect the decision-making process, since the monitored data is partially observed information about the population of SWD, which will directly affect percentage of damage of fruits due to SWD.

For adult fly captured per trap fly_males_t and $fly\_females_t$, I binned them into 3 bins, 0 for no adult flies trapped, 1 for less than or equal to 20 flies trapped in each trap on average, and 2 for greater than 20 flies trapped in each trap on average; and for larva found rate $larva_t$, I discretized it into a dummy, 0 for less than or equal to 50% and 1 for greater than 50%. And I use -99 to represent the condition that the grow has not yet monitored this season for each of the three variables. Figures 3 and 4 shows the original data for these three variables in MI 2016 data. And Figures 5 and 6 are figures of those of 2 samples from MI 2018 data.

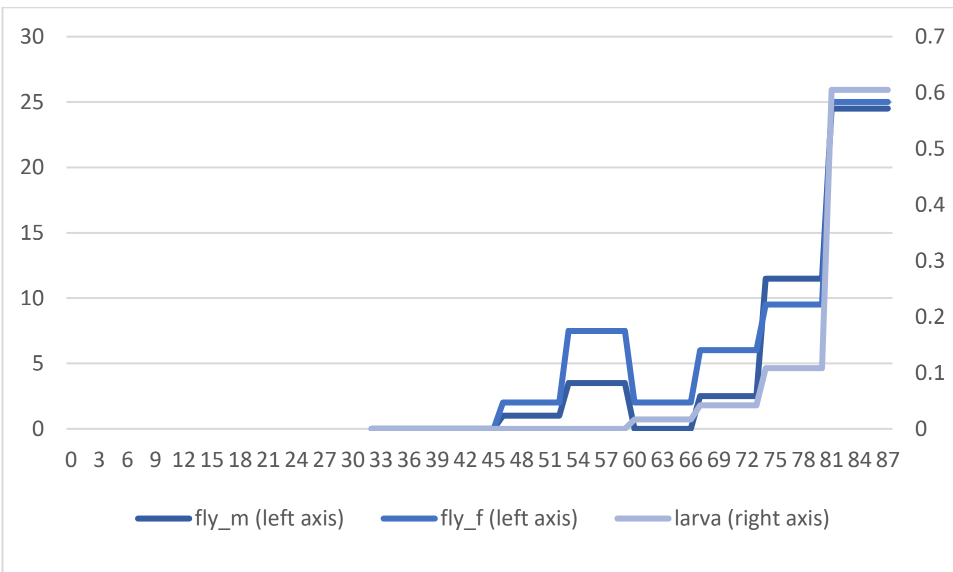

Figure 3 Monitoring Data of Grower 1 in MI 2016 Data

Note: I am not showing -99 for a better scaling.

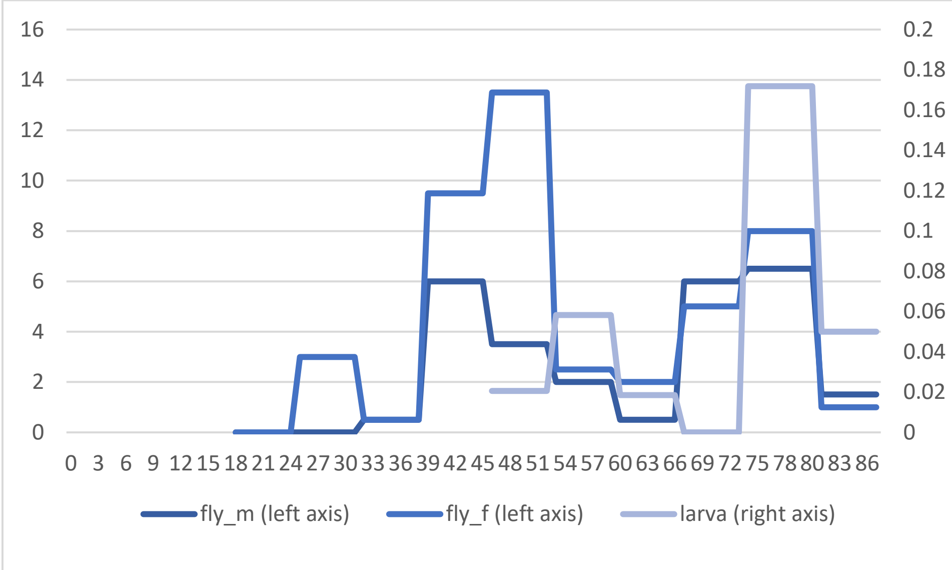

Figure 4 Monitoring Data of Grower 2 in MI 2016 Data

Note: I am not showing -99 for a better scaling.

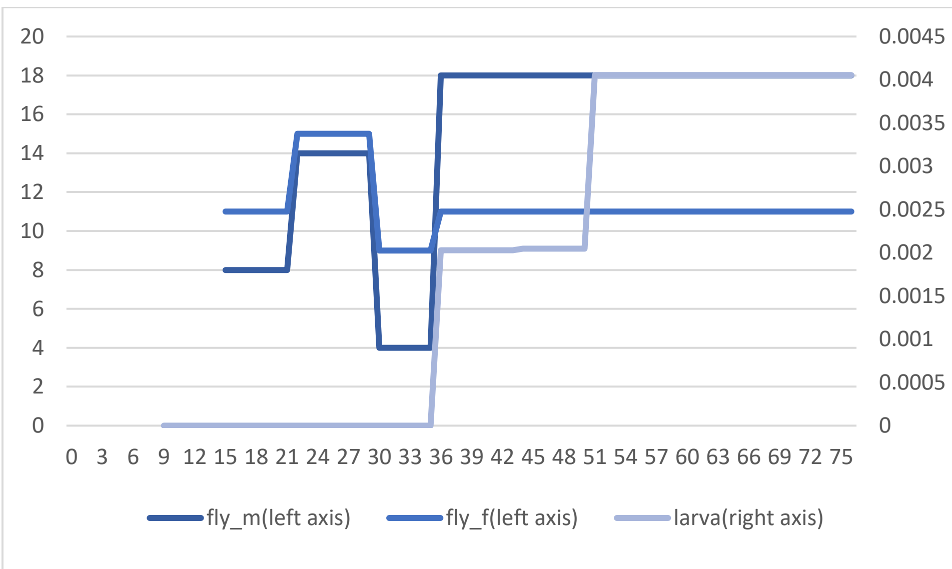

Figure 5 Monitoring Data of Grower 1 Bluecrop Cultivar in MI 2018 Data

Note: I am not showing -99 for a better scaling.

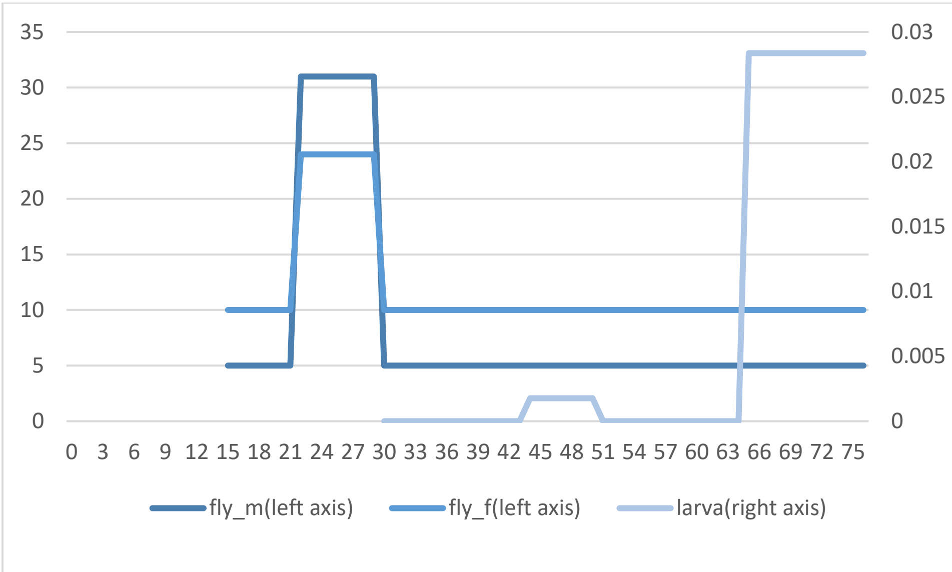

Figure 6 Monitoring Data of Grower 1 Elliott Cultivar in MI 2018 Data

Note: I am not showing -99 for a better scaling.

Tables 5 to 7 illustrate the distribution of the corresponding discretized variables in 2016 and 2018 data respectively.

Table 5 Summary of Discretized Monitoring Data (2016 MI)

Values		-99		0		1		2	
		Count	Proporsion	Count	Proporsion	Count	Proporsion	Count	Proporsion
fly_f_ discrete	Grower 1	33	35.87%	21	22.83%	28	30.43%	10	10.87%
	Grower 2	19	20.65%	14	15.22%	59	64.13%	0	0.00%
	Total	52	28.26%	35	19.02%	87	47.28%	10	5.43%
fly_m_ discrete	Grower 1	33	35.87%	14	15.22%	35	38.04%	10	10.87%
	Grower 2	19	20.65%	7	7.61%	66	71.74%	0	0.00%
	Total	52	28.26%	21	11.41%	101	54.89%	10	5.43%
larva_ discrete	Grower 1	40	43.48%	42	45.65%	10	10.87%		
	Grower 2	47	51.09%	45	48.91%	0	0.00%		
	Total	87	47.28%	87	47.28%	10	5.43%		

Table 6 Summary of Discretized Female Flies Monitoring Data (2018 MI)

Values		-99		0		1		2	
		Count	Proporsion	Count	Proporsion	Count	Proporsion	Count	Proporsion
fly_f_ discrete	**Bluecrop:**	**134**	**29.00%**	**13**	**2.81%**	**315**	**68.18%**	**0**	**0.00%**
	Grower 1	15	19.48%	0	0.00%	62	80.52%	0	0.00%
	Grower 2	77	100.00%	0	0.00%		0.00%	0	0.00%
	Grower 3	9	11.69%	13	16.88%	55	71.43%	0	0.00%
	Grower 4	15	19.48%	0	0.00%	62	80.52%	0	0.00%
	Grower 5	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Grower 6	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Jersey:	**140**	**30.30%**	**0**	**0.00%**	**222**	**48.05%**	**100**	**21.65%**
	Grower 1	15	19.48%	0	0.00%	15	19.48%	47	61.04%
	Grower 2	77	100.00%	0	0.00%		0.00%	0	0.00%
	Grower 3	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Grower 4	15	19.48%	0	0.00%	62	80.52%	0	0.00%
	Grower 5	9	11.69%	0	0.00%	21	27.27%	47	61.04%
	Grower 6	15	19.48%	0	0.00%	56	72.73%	6	7.79%
	Elliott:	**134**	**29.00%**	0	**0.00%**	**271**	**58.66%**	**57**	**12.34%**
	Grower 1	15	19.48%	0	0.00%	54	70.13%	8	10.39%
	Grower 2	77	100.00%	0	0.00%	0	0.00%	0	0.00%
	Grower 3	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Grower 4	15	19.48%	0	0.00%	21	27.27%	41	53.25%
	Grower 5	9	11.69%	0	0.00%	60	77.92%	8	10.39%
	Grower 6	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Grand Total	**408**	**29.44%**	**13**	**0.94%**	**808**	**58.30%**	**157**	**11.33%**

Table 7 Summary of Discretized Male Flies Monitoring Data (2018 MI)

Values		-99		0		1		2	
		Count	Proporsion	Count	Proporsion	Count	Proporsion	Count	Proporsion
fly_m_ discrete	**Bluecrop:**	**134**	**29.00%**	**13**	**2.81%**	**315**	**68.18%**	**0**	**0.00%**
	Grower 1	15	19.48%	0	0.00%	62	80.52%	0	0.00%
	Grower 2	77	100.00%	0	0.00%		0.00%	0	0.00%
	Grower 3	9	11.69%	13	16.88%	55	71.43%	0	0.00%
	Grower 4	15	19.48%	0	0.00%	62	80.52%	0	0.00%
	Grower 5	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Grower 6	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Jersey:	**140**	**30.30%**	**19**	**4.11%**	**154**	**33.33%**	**149**	**32.25%**
	Grower 1	15	19.48%	0	0.00%	15	19.48%	47	61.04%
	Grower 2	77	100.00%	0	0.00%		0.00%	0	0.00%
	Grower 3	9	11.69%	19	24.68%	49	63.64%	0	0.00%
	Grower 4	15	19.48%	0	0.00%	48	62.34%	14	18.18%
	Grower 5	9	11.69%	0	0.00%	27	35.06%	41	53.25%
	Grower 6	15	19.48%	0	0.00%	15	19.48%	47	61.04%
	Elliott:	**134**	**29.00%**	13	**2.81%**	**307**	**66.45%**	**8**	**1.73%**
	Grower 1	15	19.48%	0	0.00%	54	70.13%	8	10.39%
	Grower 2	77	100.00%	0	0.00%	0	0.00%	0	0.00%
	Grower 3	9	11.69%	6	7.79%	62	80.52%	0	0.00%
	Grower 4	15	19.48%	7	9.09%	55	71.43%	0	0.00%
	Grower 5	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Grower 6	9	11.69%	0	0.00%	68	88.31%	0	0.00%
	Grand Total	**408**	**29.44%**	**45**	**3.25%**	**776**	**55.99%**	**157**	**11.33%**

Since the monitoring of adult SWD and larva are serving for different purposes and adopting distinct methods, the starting date of monitoring data may be different for flies and larva, but it will be the same for female and male flies. Typically, the larva sampling starts a few weeks later then adult fly monitoring. Therefore, this can be the case that both $fly\_f\_discrete_t$, $fly\_m\_discrete_t$ are available however $larva\_discrete_t$ is -99.

These variables will evolve as functions of γ_t, $larva\_discrete_t$, $fly\_f\_discrete_t$, $fly\_m\_discrete_t$, $int\_mon\_adu_t$, $int\_mon\_lar_t$, int_ins_t, $last_t$, max_active_O, and max_active_P (or $max\_active\_mustang$ and $max\_active\_brigade$ for 2018 data) , which can be estimated by nonparametric methods, such as empirical average, which enables us to resolve the endogeneity problem. However, this may be demanding in sample size, thus if applicable, I may incorporate biological model that involves knowledge of pest population dynamics, allowing for more precise modeling of optimal pest management, which is discussed later. Therefore, these variables have independent identically conditional distributions, $\mathrm{F}_{\mathrm{larva}}(larva\_discrete_{t+1} | g_t, \vec{x}_t, \vec{z}_t)$, $\mathrm{F}_{\mathrm{fly\_f}}(fly\_f\_discrete_{t+1} | g_t, \vec{x}_t, \vec{z}_t)$ and $\mathrm{F}_{\mathrm{fly\_}m}(fly\_m\_discrete_{t+1} | g_t, \vec{x}_t, \vec{z}_t)$ respectively.

d) The most recent spray decision, $last_t$.

These are lagged terms of insecticide application action variable and their transition matrix are deterministic given by the following relationship:

$$last_t = \begin{cases} 1\text{ , if } \left(c_{t-1} = 1\right) \vee \left(c_{t-1} = 0 \wedge last_{t-1} = 1\right) \\ 2\text{ , if } \left(c_{t-1} = 2\right) \vee \left(c_{t-1} = 0 \wedge last_{t-1} = 2\right). \\ 0\text{ , otherwise} \end{cases}$$

Here, if the grower has not yet applied any insecticide this season, then I code them all as “0” for “have not applied any insecticide”.

They are included since they may affect the insecticide category choice if the grower considers insecticide rotation to prevent resistance development. Although the implementation of rotation may escalate cost of application, but I hypothesized that farmers would like to sacrifice a little in the current insecticide cost for not losing the cheaper options forever.

3.4.4. Empirical Strategy for MI 2016 Data

Since no yield or lost due to SWD damage data is available now, I consider a per-period payoff function consists of two parts, one visible cost part, and two invisible cost part, including the risk part I composed of interval variables and the risk part II composted of monitoring results, to capture the risk aversion characteristics of the growers.

To initiate, I assume that there is a cost incurred for monitoring and insecticide application for all time period depending on the actions taken expressed by

$$\begin{aligned} \text{cost}(a_{it}, b_{it}, c_{it} \,|\, \vec{x}_t) = -cost\_ins(c_{it}) - cost\_adu(a_{it}) - cost\_lar(b_{it}) \\ -\theta_1 \times l\_same_{it} \times c\_last\_dummy_{it} \end{aligned} \qquad (1)$$

If the growers care about the rotation of insecticides at all, there will exist an invisible cost representing the risk of elevating the insecticide resistance when the insecticide choice is the same as the last spraying decision. Thus, I incorporate this nuance with the last term in Equation (1).

If the grower does not spray insecticide on day *t* (i.e., $c_{it} = 0$), the grower risks SWD infestation. Firstly, consider the risk I part which is the interval variable part of the cost function, the negative of the per-period payoff function.

As mentioned before, it is intuitive that the grower will at dramatically higher risk of infestation when the intervals are greater than the maximum days of activity of the insecticides, so there may be a jumping discontinuity in the effects of interval variables on the utility function. Additionally, the growers may also apply insecticides according to the calendar instead of the monitoring data, in which case they may apply insecticides whenever the interval variables being exactly the maximum days of activity. In sight of this, we define two variables int_ratio_under and int_ratio_over out of interval since last insecticide application as below.

$$int\_ratio_{it} = \begin{cases} \dfrac{int\_ins_{it}}{\max\_active(last_{it})}, & \text{if } last_{it} \neq -99 \\ T, & \text{if } last_{it} = -99 \end{cases}$$

$$int\_ratio\_over_{it} = \begin{cases} 0, & \text{if } int\_ratio_{it} < 1 \\ int\_ratio_{it}, & \text{if } int\_ratio_{it} \geq 1 \end{cases}$$

$$int\_ratio\_under_{it} = \begin{cases} int\_ratio_{it}, & \text{if } int\_ratio_{it} < 1 \\ 0, & \text{if } int\_ratio_{it} \geq 1 \end{cases}$$

And we discretize the monitoring interval variables as below to form our second function specification.

$$int\_mon\_adu\_discrete_t = \begin{cases} 0, & \text{if } 1 \leq int\_mon\_adu < max\_active \\ 1, & \text{if } int\_mon\_adu = max\_active \\ 2, & \text{if } int\_mon\_adu > max\_active \vee int\_mon\_adu = 99 \end{cases},$$

$$int\_mon\_lar\_discrete_t = \begin{cases} 0, & \text{if } 1 \leq int\_mon\_lar < max\_active \\ 1, & \text{if } int\_mon\_lar = max\_active \\ 2, & \text{if } int\_mon\_lar > max\_active \vee int\_mon\_lar = 99 \end{cases},$$

where max_active is the maximum days of activity of the last insecticide applied.

Hence, the risk I part is sketched as below.

$$\text{risk}^{\text{I}}\left(\gamma_{it}, \vec{x}_{it}, \vec{z}_{it}; \boldsymbol{\theta}\right) = -\left[c_{it} = 0\right] \times \left\{ \begin{array}{l} \theta_2 \times int\_mon\_adu\_discrete_{it} + \\ \theta_3 \times int\_mon\_lar\_discrete_{it} + \\ \left[last_{it} \neq -99\right] \times \left(\begin{array}{l} \theta_4 \times int\_ratio\_under_{it} + \\ \theta_5 \times int\_ratio\_over_{it} \end{array} \right) + \\ \left[last_{it} = -99\right] \times \theta_5 \times T \end{array} \right\} \quad (2)$$

Then consider the risk II part of the equation. For the Michigan 2016 panel data, there are only 2 growers, so it does not make sense to capture an unobserved heterogeneity using an E-M algorithm to estimate its distribution. Instead, with only 2 growers, any unobserved heterogeneity would be fully captured simply by having a different constant for each grower, which is a grower fixed effect.[2] Therefore, I adopt a set of dummy variables for each of the two growers, $\eta^{(1)}$ and $\eta^{(2)}$, where

$$\eta^{(i)} = \begin{cases} 1, & \text{if the observation belongs to grower i} \\ 0, & \text{otherwise} \end{cases} \qquad (i = 1, 2).$$

Additionally, I assume that if the grower has not yet monitored, the risk is as if it the measurements were all the worst-case scenario, which assembles the risk aversion of growers. In this case, values of discretized adult monitoring variables will be 2 and value of discretized larva monitoring variable will be 1. For concreteness, with the assumption of the worst-case risk scenario if grower has not yet monitored, I define

$$fly\_m\_disc\_expected_t = \begin{cases} fly\_m\_discrete_t, \text{ if } fly\_m\_discrete_t \neq -99 \\ \quad 2 \quad , \ otherwise \end{cases},$$

$$fly\_f\_disc\_expected_t = \begin{cases} fly\_f\_discrete_t, \text{ if } fly\_f\_discrete_t \neq -99 \\ \quad 2 \quad , \ otherwise \end{cases},$$

$$larva\_disc\_expected_t = \begin{cases} larva\_discrete_t, \text{ if } larva\_discrete_t \neq -99 \\ \quad 1 \quad , \ otherwise \end{cases},$$

Lastly, to capture the feature that the relationship between the pest population and the actions may be non-linear, since there might be some threshold above which the farmers will abandon the fruits and not apply insecticides anymore in that growing season is a good one, I the fully non-parametric, with a different coefficient on each bin of pest monitoring variables in th risk part.

Thus, I have construed the risk part as below, which only exists when the choice of action this time period is 0.

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[2] This is probably why we are having trouble identifying distribution of the unobserved heterogeneity when using the E-M algorism, and that the best specifications seem to have probability of high susceptibility either as 0.5 (i.e., different constant for each grower) or close to 1 (i.e., same constant for each grower). For more details about this method, please refer to section 3.4.5.

$$\text{risk}^{\text{II}}\left(\gamma_{it},\vec{x}_{it},\vec{z}_{it};\boldsymbol{\theta}\right) = -\left[c_{it}=0\right]\times\left\{\begin{array}{l}\theta_6\times\left[fly\_m\_disc\_expected_{it}=1\right]+\\ \theta_7\times\left[fly\_m\_disc\_expected_{it}=2\right]+\\ \theta_8\times\left[fly\_f\_disc\_expected_{it}=1\right]+\\ \theta_9\times\left[fly\_f\_disc\_expected_{it}=2\right]+\\ \theta_{10}\times\left[lar\_disc\_expected_{it}=1\right]+\\ \theta_{11}\times\eta_i^{(1)}+\\ \theta_{12}\times\eta_i^{(2)}\end{array}\right\} \tag{3}$$

Therefore, our final version of specification for the per-period payoff in 2016 model is illustrated as below.

$$\begin{aligned}\pi_0\left(\gamma_{it},\vec{x}_{it},\vec{z}_{it};\boldsymbol{\theta}\right) = &-cost\_ins(c_{it})-cost\_adu(a_{it})-cost\_lar(b_{it})\\ &-\theta_1\times l\_same_{it}\times c\_last\_dummy_{it}\\ &-\left[c_{it}=0\right]\times\left\{\begin{array}{l}\theta_2\times int\_mon\_adu\_discrete_{it}+\\ \theta_3\times int\_mon\_lar\_discrete_{it}+\\ \left[last_{it}\neq -99\right]\times\left(\begin{array}{l}\theta_4\times int\_ratio\_under_{it}+\\ \theta_5\times int\_ratio\_over_{it}\end{array}\right)+\\ \left[last_{it}=-99\right]\times\theta_5\times T\end{array}\right\}\\ &-\left[c_{it}=0\right]\times\left\{\begin{array}{l}\theta_6\times\left[fly\_m\_disc\_expected_{it}=1\right]+\\ \theta_7\times\left[fly\_m\_disc\_expected_{it}=2\right]+\\ \theta_8\times\left[fly\_f\_disc\_expected_{it}=1\right]+\\ \theta_9\times\left[fly\_f\_disc\_expected_{it}=2\right]+\\ \theta_{10}\times\left[lar\_disc\_expected_{it}=1\right]+\\ \theta_{11}\times\eta_i^{(1)}+\\ \theta_{12}\times\eta_i^{(2)}\end{array}\right\}\\ &+\varepsilon_{it}(\gamma_{it})\end{aligned} \tag{4}$$

Here, $[\bullet]$ is the Iverson bracket, which converts any logical proposition into a number that is 1 if the proposition is satisfied, and 0 otherwise, i.e.,

$$[\text{P}]=\begin{cases}1, \text{ if P is true;}\\ 0, \text{ otherwise.}\end{cases}$$

We define $\boldsymbol{\theta}=(\theta_1,\theta_2,...,\theta_{12})'$ are the parameters to be estimated. $\varepsilon_{it}(\gamma_{it})$ is the private shock to grower i at time t, which is assumed to be independent identically distributed across all agents, time periods and actions. We assume the private shocks ε_{it} follow i.i.d. extreme value distribution conditional on decision γ_{it}.

And I hypothesize that the parameters here, θ_1 to θ_{10}, are statistically significant and positive, implying that the growers are taking actions, with the guidance of the monitoring data, and with the consideration of insecticide resistance and sustainability.

3.4.5. Empirical Strategy for MI 2018 Data

Firstly, I introduce a new set of variables, dummies for the presence of blueberry variety X on the plot, δ_X, where

$$\delta_X = \begin{cases} 1, & \text{if blueberry type X is on the plot} \\ 0, & \text{if blueberry type X is not on the plot} \end{cases},$$

$$X \in \{\text{Blue Crop, Jersey, Elliott}\}.$$

These variables enable us to incorporate variety fixed effect which controls for the preference towards different harvesting time.

Secondly but more importantly, I will allow for unobserved heterogeneity in the susceptibility to SWD of the blueberries across different farms in this model. The heterogeneity can raise from different aspects, for instance, as below:

a) Different level of susceptibility to SWD of the blueberries across different farms. This may due to different geographic conditions of different farms. As reported by Rufus Isaacs et al. (2015), presence of honeysuckle near fields is a predictor of more activity from SWD. And they also observed higher catches in traps adjacent to fields where they remain wet longer, or adjacent to creeks.

b) Methods taken before the data available can affect the farms' condition in SWD infection. For instance, improper use of insecticides or a lack in insecticide rotation may lead to higher resistance and higher vulnerability of the SWD population specific to that neighborhood.

c) The growers are making the decisions depending on their perception of whether the neighbor is actively managing their fields and the infection situation of the cultivars harvested earlier, which cannot be observed by the econometricians. They may treat fields near infested fields or careless neighbors as susceptible fields and apply more insecticides.

Here, for simplicity, I adopt only one unobserved state variable, v, representing the susceptibility to SWD of the blueberry farms. It is discretized into two different levels, 0 (low susceptibility level) and 1 (high susceptibility level).

The only difference in the per-period payoff setting is the risk II function. The version for the MI 2018 data is shown as below.

$$\text{risk}^{\text{II}}\left(\gamma_{it}, \vec{x}_{it}, \vec{z}_{it}; \boldsymbol{\theta}\right) = -\left[c_{it} = 0\right] \times \left\{ \begin{array}{l} \theta_6 \times \left[fly\_m\_disc\_expected_{it} = 1 \right] + \\ \theta_7 \times \left[fly\_m\_disc\_expected_{it} = 2 \right] + \\ \theta_8 \times \left[fly\_f\_disc\_expected_{it} = 1 \right] + \\ \theta_9 \times \left[fly\_f\_disc\_expected_{it} = 2 \right] + \\ \theta_{10} \times \left[lar\_disc\_expected_{it} = 1 \right] + \\ \theta_{11} \times \delta_{Bluecrop} + \\ \theta_{12} \times \delta_{Jersey} + \\ \theta_{13} \times \delta_{Elliott} + \\ \theta_{14} \times v \end{array} \right\} \quad (5)$$

Therefore, our final version of specification for the per-period payoff in 2018 model is illustrated as below.

$$
\begin{aligned}
\pi_0\left(\gamma_{it}, \bar{x}_{it}, \bar{z}_{it}; \boldsymbol{\theta}\right) = & -cost\_ins(c_{it}) - cost\_adu(a_{it}) - cost\_lar(b_{it}) \\
& -\theta_1 \times l\_same_{it} \times c\_last\_dummy_{it} \\
& -\left[c_{it}=0\right] \times \left\{ \begin{array}{l} \theta_2 \times int\_mon\_adu\_discrete_{it} + \\ \theta_3 \times int\_mon\_lar\_discrete_{it} + \\ \left[last_{it} \neq -99\right] \times \left(\begin{array}{l} \theta_4 \times int\_ratio\_under_{it} + \\ \theta_5 \times int\_ratio\_over_{it} \end{array} \right) + \\ \left[last_{it} = -99\right] \times \theta_5 \times T \end{array} \right\} \\
& -\left[c_{it}=0\right] \times \left\{ \begin{array}{l} \theta_6 \times \left[fly\_m\_disc\_expected_{it} = 1\right] + \\ \theta_7 \times \left[fly\_m\_disc\_expected_{it} = 2\right] + \\ \theta_8 \times \left[fly\_f\_disc\_expected_{it} = 1\right] + \\ \theta_9 \times \left[fly\_f\_disc\_expected_{it} = 2\right] + \\ \theta_{10} \times \left[lar\_disc\_expected_{it} = 1\right] + \\ \theta_{11} \times \delta_{Bluecrop} + \\ \theta_{12} \times \delta_{Jersey} + \\ \theta_{13} \times \delta_{Elliott} + \\ \theta_{14} \times v \end{array} \right\} \\
& + \varepsilon_{it}(\gamma_{it})
\end{aligned} \tag{6}
$$

Here, $[\bullet]$ is the Iverson bracket, which converts any logical proposition into a number that is 1 if the proposition is satisfied, and 0 otherwise, i.e.,

$$
[\text{P}] = \begin{cases} 1, & \text{if P is true;} \\ 0, & \text{otherwise.} \end{cases}
$$

We define $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_{14})'$ are the parameters to be estimated. $\varepsilon_{it}(\gamma_{it})$ is the private shock to grower i at time t, which is assumed to be independent identically distributed across all agents, time periods and actions. We assume the private shocks ε_{it} follow i.i.d. extreme value distribution conditional on decision γ_{it}.

And we hypothesize that the parameters, θ_1 to θ_{10} and θ_{14}, are statistically significant and positive, implying that the growers are taking actions, with the guidance of the monitoring data, with the consideration of insecticide resistance and sustainability, and with the recovered observation of unobserved heterogeneity.

With these in mind, we can now give the sequence problem and Bellman equation in this case. The decisions made in each period depend only on the current values of the state variables $\overrightarrow{x_t}, \overrightarrow{z_t}, \varepsilon_t, v$.[3] The decision process can then be described as a policy function $\varphi\left(\overrightarrow{x_t}, \overrightarrow{z_t}, \varepsilon_t, v\right)$. A sequence of decision rules $\sigma_T = \left(\varphi_0, \varphi_1, \ldots, \varphi_T\right)$ is a decision policy. The optimal policy is the one that maximizes the grower's discounted present value of the entire stream of per-period payoff, as given by the following dynamic optimization sequence problem:

$$\max_{\{\gamma_t \in \Gamma\}_{t=0}^{T}} E\left\{\sum_{t=0}^{T+1} \beta^t \pi\left(\gamma_t, \vec{x}_t, \vec{z}_t, v, \varepsilon_t; \boldsymbol{\theta}\right) \middle| \vec{x}_0, \vec{z}_0, v\right\}$$

The value function for each time t is given by the following Bellman equation:

$$V_t\left(\vec{x}, \vec{z}, \varepsilon, v\right) = \max_{\gamma \in \Gamma} \left\{\pi\left(\gamma, \vec{x}, \vec{z}, \varepsilon, v; \boldsymbol{\theta}\right) + \beta \mathrm{E}\left[V_{t+1}\left(\vec{x}', \vec{z}', \varepsilon', v\right) \middle| \vec{x}, \vec{z}, \varepsilon, v; \boldsymbol{\theta}\right]\right\}, t = 0, \ldots, T$$

where $\vec{x}, \vec{z}, \varepsilon$ are the current values, and $\vec{x}', \vec{z}', \varepsilon'$ are the future values of the variables. Producers observing the current state of $\left(\vec{x}, \vec{z}, \varepsilon\right)$ will choose action $\gamma \in \Gamma$ to maximize the current period payoff plus the discounted value of the expected future value function. The dynamic programming problem can be solved backwards starting with harvest period T+1, when the per-period payoff is assumed to be 0.

We use a daily discount factor of $\beta_{daily} = \dfrac{\exp(\ln(\beta_{annual}))}{T+1}$, which yields an annual discount factor of $\beta_{annual} = 0.9$ over the $(T+1)$-day finite horizon.

---

[3] From here on, to make it concise, we only index the variables with t, considering it as identical and independent finite horizon dynamic decision progress in each growing season for each variety the grower plants.

CHAPTER 4

ECONOMETRIC ESTIMATION APPROACH

4.1.Estimation Approach for 2016 Model

4.1.1. Estimating Continuation Value by Backward Iteration

The vector of parameters to be estimated is $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_{14})'$.

The dynamic structural model set up above cannot identify the discounting factor β, thus we would like to specify two different values for the discount factor, $\beta_{daily} = 0.999$ for dynamic scenario and $\beta_{daily} = 0.95$ for myopic scenario, and conduct a likelihood ratio test between these two situations to determine which type of process has higher predictability for the decisions observed in the data. We may find other variables to identify the discount factor later to check for the robustness of the results.

We assume the private shocks ε_t follow i.i.d. extreme value distribution conditional on decision γ_t.

And we also assume conditional independence as below:

$$\Pr(\overrightarrow{x_{t+1}}, \overrightarrow{z_{t+1}}, \varepsilon_{t+1} \left| \overrightarrow{x_t}, \overrightarrow{z_t}, v, \gamma_t; \boldsymbol{\theta}) = \Pr(\overrightarrow{x_{t+1}}, \overrightarrow{z_{t+1}} \right| \overrightarrow{x_t}, \overrightarrow{z_t}, v, \gamma_t; \boldsymbol{\theta}) \Pr(\varepsilon_{t+1} | \overrightarrow{x_t}, \overrightarrow{z_t}, v, \boldsymbol{\theta})$$

Let $\pi_0\left(\gamma, \vec{x}, \vec{z}, v; \boldsymbol{\theta}\right)$ denote the deterministic component of the per-period payoff, which is assumed to be linearly separable from the stochastic component $\varepsilon\left(\gamma\right)$, the Bellman equation can then be rewritten as:

$$V_t\left(\vec{x}, \vec{z}, \varepsilon, v\right) = \max_{\gamma \in \Gamma} \left\{ \pi_0\left(\gamma, \vec{x}, \vec{z}, v; \boldsymbol{\theta}\right) + \varepsilon\left(\gamma\right) + \beta \mathrm{E}\left[V_{t+1}\left(\vec{x}', \vec{z}', \varepsilon', v\right) \middle| \vec{x}, \vec{z}, \gamma, v; \boldsymbol{\theta} \right] \right\}, t = 0, \ldots, T$$

The continuation value $\mathrm{E}\left[V_{t+1}\left(\vec{x}', \vec{z}', \varepsilon', v\right) \middle| \vec{x}, \vec{z}, \gamma, v; \boldsymbol{\theta} \right]$ is the expectation of the value function next period taken over error term ε and all possible states of next period, conditional on this period's states and actions taken $\left(\vec{x}, \vec{z}, v, \gamma\right)$. We denote the continuation value as $U_t\left(\vec{x}_t, \vec{z}_t, \gamma_t, v; \boldsymbol{\theta}\right) = \mathrm{E}\left[V_{t+1}\left(\vec{x}', \vec{z}', \varepsilon', v\right) \middle| \vec{x}, \vec{z}, \gamma, v; \boldsymbol{\theta} \right]$.

Then the Bellman equation becomes

$$V_t(\vec{x}, \vec{z}, \varepsilon, v) = \max_{\gamma \in \Gamma} \{ \pi_0(\gamma, \vec{x}, \vec{z}, v; \boldsymbol{\theta}) + \varepsilon(\gamma) + \beta U_t(\gamma, \vec{x}, \vec{z}, v; \boldsymbol{\theta}) \}, t = 0, ..., T$$

Thus, the choice probability can be expressed as (P_m used to denote the choice probability of taking the m-th valid action $\gamma_m \in \Gamma$)

$$P_m\left(\gamma_t \middle| \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right) = \frac{\exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right)\right]}{\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right)\right]}$$

The likelihood function for the entire sample is:

$$L(\boldsymbol{\theta}) = \prod_{t=0}^{T} \frac{\exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right)\right]}{\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right)\right]}$$

Since this is a finite horizon problem, $U_t\left(\vec{x}_t, \vec{z}_t, \gamma_t, v; \boldsymbol{\theta}\right)$ can be solved by backwards iteration in each step when finding the maximum likelihood estimator by iteration. We iterate backwards from the terminal condition $U_T\left(\vec{x}_T, \vec{z}_T, \gamma_T, v; \boldsymbol{\theta}\right) = 0$

For the bio state variables *(fly_m_discrete, fly_f_discrete, larva_discrete)*, I estimate their transition density non-parametrically using empirical averages as a start. In particular, the distribution of $(fly\_m\_discrete_{t+1},\ fly\_f\_discrete_{t+1},\ larva\_discrete_{t+1})$ next period depends on $(fly\_m\_discrete_t,\ fly\_f\_discrete_t,\ larva\_discrete_t)$ this period and on the action chosen this period. Later I may use a (possibly empirically estimated) population growth model for SWD in place of this non-parametric transition density.

To magnify the different influences of interval length that are greater than the maximum days of activity of the insecticides on the bio state variables, since we believe that the infestation risk will increase if the crops are no longer effectively protected by the insecticides, I estimate 2 separate transition densities, *M_under* for when $int\_ins < max\_active$, and *M_over* for when $int\_ins \geq max\_active$ which includes the case when we have not sprayed yet this season.

4.1.2. Estimating Parameters via Finite Horizon DP Nested in MLE

To estimate the coefficients in the per-period payoff function, we will use MLE with finite horizon dynamic programming (DP) problem nested in the likelihood function

$$L(\boldsymbol{\theta}) = \prod_{t=0}^{T} \frac{\exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right)\right]}{\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t, v; \boldsymbol{\theta}\right)\right]}.$$

This can help to deal with the endogeneity caused by simultaneous equation problem between the action variables and the biological state variables. To be more specific, the actions the growers take are based on what they observe by monitoring the adult flies and the larva. However, the monitoring data, which are the biological state variables, are simultaneously influenced by the actions taken by the growers. Therefore, there will be two simultaneous equations with correlated error terms. To solve this problem, we introduce the non-parametric estimation of the transition density and the structural model framework. That is to embed the finite horizon sequence problem estimated by backward iteration demonstrated above into the MLE method.

4.1.3. Estimating Standard Errors by Analytical Derivation

Since there are only two samples in the MI 2016 data, we cannot really calculate the standard error with bootstrapping. Thus, we choose to calculate the variance matrix analytically for 2016 model. For the analytical derivation, please refer to Appendix I.

4.2.Estimation Approach for 2018 Model

4.2.1. Estimating Coefficients Using EM Algorithm

To incorporate the unobserved heterogeneity of susceptibility to SWD, we adopt the Expectation Maximization algorithm (EM algorithm) to estimate parameters $\boldsymbol{\theta}$ as well as the probability of being in unobserved high susceptibility state, P_v.

Denote the entire vector of observations of actions and states, respectively, over all days for farm-year combination $n \in \{1, 2, \ldots, N\}$ as γ_n and $\vec{x}_{nt}, \vec{z}_{nt}$, q_{nv} is defined as the conditional probability that farm n is in unobserved state v:

$$q_{nv} = \Pr(v \mid \gamma_n, \vec{x}_n, \vec{z}_n; \boldsymbol{\theta}, P_v, P_m)$$

1) Start with an initial guess for the first iteration $\boldsymbol{\theta}^{(1)}, P_v^{(1)}, P_m^{(1)}$.

2) In each m-th iteration, run through the following E-step and M-step:

E-step: Calculate conditional probability q_{nv} of each observation being in unobserved state.

Step 1: update q_{nv}.

Form likelihood for grower-time observation:

$$l\left(\gamma_{nt} \middle| \vec{x}_{nt}, \vec{z}_{nt}, v; P_m^{(k)}, \boldsymbol{\theta}^{(k)}\right) = \frac{\exp\left[\pi_0\left(\gamma_{nt}, \vec{x}_{nt}, \vec{z}_{nt}, v; \boldsymbol{\theta}^{(k)}\right) + \beta U_{nt}\left(\gamma_{nt}, \vec{x}_{nt}, \vec{z}_{nt}, v; \boldsymbol{\theta}^{(k)}\right)\right]}{\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_{nt}, \vec{x}_{nt}, \vec{z}_{nt}, v; \boldsymbol{\theta}^{(k)}\right) + \beta U_{nt}\left(\tilde{\gamma}_{nt}, \vec{x}_{nt}, \vec{z}_{nt}, v; \boldsymbol{\theta}^{(k)}\right)\right]}$$

From Bayes' Rule, update q_{nv}:

$$q_{nv}^{(k+1)} = \frac{\mathrm{P}_{\mathrm{v}}^{(k)} \prod_{t=0}^{T} l\left(\gamma_{nt} \middle| \vec{x}_{nt}, \vec{z}_{nt}, v; P_m^{(k)}, \boldsymbol{\theta}^{(k)}\right)}{\sum_{v'=0}^{1} \mathrm{P}_{\mathrm{v'}}^{(k)} \prod_{t=0}^{T} l\left(\gamma_{nt} \middle| \vec{x}_{nt}, \vec{z}_{nt}, v'; P_m^{(k)}, \boldsymbol{\theta}^{(k)}\right)}$$

Step 2: update P_v.

$$\mathrm{P}_{\mathrm{v}}^{(k+1)} = \frac{1}{N} \sum_{n=1}^{N} q_{nv}^{(k+1)}$$

Step 3: update $P_m\left(\gamma \middle| \vec{x}, \vec{z}, v; \boldsymbol{\theta}\right)$

$$P_m^{(k+1)}\left(\vec{x}_{nt}, \vec{z}_{nt}, v\right) = l\left(\gamma_{nt} = \gamma_m \middle| \vec{x}_{nt}, \vec{z}_{nt}, v; P_m^{(k)}, \boldsymbol{\theta}^{(k)}\right)$$

M-step: Treat unobserved state as observed, using conditional probability q_{nv} of unobserved state as weights.

Step 4: Taking $q_{nv}^{(k+1)}$ and $P_m^{(k+1)}\left(\vec{x}_{nt}, \vec{z}_{nt}, v\right)$ as given, solve for $\boldsymbol{\theta}^{(k+1)}$.

$$\boldsymbol{\theta}^{(k+1)} = \arg\max_{\boldsymbol{\theta}} \sum_{n=1}^{N} \sum_{v=0}^{1} \sum_{t=0}^{T} q_{nv}^{(k+1)} \ln l\left(\gamma_{nt} \middle| \vec{x}_{nt}, \vec{z}_{nt}, v; P_m^{(k)}, \boldsymbol{\theta}^{(k)}\right)$$

Iterate through step 1-4, i.e., iterate through E-step and M-step until convergence to get the estimation of the parameters and probabilities.

4.2.2. Estimating Standard Errors via Bootstrapping

We will estimate standard errors of the parameters θ using a bootstrap. Grower-years are randomly drawn from the data set with replacement to generate 100 independent panels each with the same number of grower-years as in the original data set. The structural model is run on each of the panels. The standard errors are then formed by taking the standard deviation of the estimates from each of the random samples.

CHAPTER 5

RESULTS

Table 8 presents our preliminary results for the parameter estimates for the 2016 model. The results already bring us some insights in the optimal SWD management strategy in the IPM program. Later with more thoroughly exploration of data following the empirical approaches developed in the previous chapters, we will be able to establish a framework assisting in generation of most beneficial decisions automatically.

As illustrated in Table 8, θ_1 is negative, which implies that when applying the same insecticide as last application, the utility will be higher than that when doing insecticide rotation. This may because that the growers prefer using the same insecticide and may not be aware of or care about the potential for insecticide resistance that may result from using the same insecticide over and over again.

θ_2 is negative, but θ_3 is positive. We may expect the interval variable of adult fly monitoring to behave similar to the that of larva monitoring, but here we detect a discrepancy. This may due to the different purpose of doing these two different kinds of monitoring in IPM. As mentioned above, the growers or extension educators mostly start adult fly monitoring several weeks before the larva monitoring to detect a early alarm for SWD infestation. On the other hand, the larva monitoring results will be directly and closely related whether the fruit is contaminated or not. Thus, the growers will abandon fly monitoring in later season, and totally rely on the larva monitoring. So, it is reasonable to have a positive θ_3, which means smaller intervals between larva monitoring will ensure lower risk, thus brings higher utility.

θ_4 and θ_5 are both positive, which suggests that, as expected, a longer interval since the last spray, relative to the maximum number days the last insecticide sprayed is effective, increases the risk and lowers utility.

θ_6 and θ_9 are all positive, which is the same direction as we expected. What's more, comparing them with each other, we could find that the growers consider the female adult observations more seriously than the male adult observations. This is intuitive, since only female flies are able to lay eggs into the ripening fruits, which is the main reason of the contamination. Additionally, the results also imply that the growers consider it more significant when they start to observe some female flies (when $fly\_f\_discrete = 1$), which coincides with the purpose of maintaining adult fly trapping.

The only conflict with our intuition is that the sign of the coefficient on larva monitoring is negative. One possible explanation can be that once the growers detect large number of larvae in the sampling fruits, they regard it as a signal of serious contamination, and will no longer invest any more money or energy to that plot. This is partially reasonable, since the processors indicate that they will not purchase any fresh blueberries from the grower once they detect any larvae in the commodity from that grower.

Table 8 Preliminary Results for 2016 Model

Coefficients		Estimates
Theta(1)	same insecticide as last application	-5.262
Theta(2)	int_mon_adu_discrete	-6.126
Theta(3)	int_mon_lar_discrete	6.122
Theta(4)	int_ins_ratio if [int_ins_ratio <1]	0.068
Theta(5)	int_ins_ratio if [int_ins_ratio>=1]	0.001
Theta(6)	[fly_m_discrete =1] (some)	0.118
Theta(7)	[fly_m_discrete =2] (many)	5.819
Theta(8)	[fly_f _discrete =1] (some)	8.453
Theta(9)	[fly_f_discrete =2] (many)	5.819
Theta(10)	[lar _discrete = 1] (high)	-3.074
Theta(11)	dummy for grower 1	103.61
Theta(12)	dummy for grower 2	46.86
log likelihood		**-1660.22**

CHAPTER 6

CONCLUSION

For my Masters thesis, I analyze the management of Spotted Wing Drosophila (SWD) in Michigan blueberry using a dynamic structural econometric model. The Spotted Wing Drosophila (SWD) is a vinegar fly of East Asian origin that can cause damage to many fruit crops. I develop a dynamic structural model to study the SWD management decisions of growers of Michigan highbush blueberry regarding fly and larva monitoring and insecticide application. I apply my dynamic structural econometric model to a detailed data set I have collected and constructed of daily decisions of blueberry growers in Michigan.

Our research questions include the following. In the context of SWD management, what is the best timing strategy to apply insecticide and which insecticide to use conditional on the monitoring information? Do growers worry about the potential for developing insecticide resistance?

In future work, we will use the structural model to determine whether the farmers are making dynamically optimal decisions or are discounting the future too much by changing different discount factor values and testing for the predictability of the model. We will use parameters estimated from our structural model to conduct counterfactual analysis.

In an extension to our dynamic structural model, we will allow for unobserved heterogeneity, which enables us to estimate the distribution of unobserved susceptibility as well as the effects of varietal susceptibility on payoffs.

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APPENDIX I ANALYTICAL STANDARD ERROR DERIVATION

For an MLE estimator, the variance of the vector theta is given by the inverse of the information matrix, where the information matrix is the negative of the expected value of the Hessian and the Hessian is the matrix of the second derivatives of the log likelihood with respect to the parameters. By getting the Hessian as below, we can finally calculate the standard error matrix analytically.

Define the continuation value to be

$$U_t\left(\vec{x}_t, \vec{z}_t, \gamma_t, v; \boldsymbol{\theta}\right) = \mathrm{E}\left[V_{t+1}\left(\vec{x}', \vec{z}', \varepsilon', v\right) \middle| \vec{x}, \vec{z}, \gamma, v; \boldsymbol{\theta}\right]$$

So,

$$\begin{aligned} U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) &= E_\varepsilon\left[\max_m E_{\vec{s}'}\left(U_{t,m}\right)\right] \\ &= E_{\vec{x}_{t+1}}\left[\log\left\{\sum_{m=1}^{num\_action} \exp\left[\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) | \vec{x}_{t+1}\right]\right]\right\} | \gamma_t, \vec{x}_t, \vec{z}\right] \end{aligned}$$

Then, the likelihood function is given by

$$L(\boldsymbol{\theta}) = \prod_{g,t} \frac{\exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]}{\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]}$$

So,

$$\ln\left(L(\boldsymbol{\theta})\right) = \sum_{g,t}\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) - \ln\left(\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\right)\right]$$

Thus,

$$\frac{\partial \ln\left(L(\boldsymbol{\theta})\right)}{\partial \boldsymbol{\theta}} = \sum_{g,t}\left[\frac{\partial}{\partial \boldsymbol{\theta}}\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta \frac{\partial}{\partial \boldsymbol{\theta}} U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) - \frac{\sum_{\tilde{\gamma}_t} \frac{\partial}{\partial \boldsymbol{\theta}} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]}{\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]}\right]$$

$$= \begin{bmatrix} \sum_{g,t}\left[\frac{\partial}{\partial \theta_1}\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta \frac{\partial}{\partial \theta_1} U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) - \frac{\sum_{\tilde{\gamma}_t} \frac{\partial}{\partial \theta_1} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]}{\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]}\right] \\ \vdots \\ \sum_{g,t}\left[\frac{\partial}{\partial \theta_{12}}\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta \frac{\partial}{\partial \theta_{12}} U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) - \frac{\sum_{\tilde{\gamma}_t} \frac{\partial}{\partial \theta_{12}} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]}{\sum_{\tilde{\gamma}_t} \exp\left[\pi_0\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\tilde{\gamma}_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]}\right] \end{bmatrix}$$

Therefore, we have

$$
\begin{aligned}
\frac{\partial^2 \ln\left(L(\boldsymbol{\theta})\right)}{\partial\boldsymbol{\theta}\,\partial\boldsymbol{\theta}'} &= \sum_{g,t}\left\{\begin{array}{l}
\frac{\partial^2}{\partial\theta\,\partial\theta'}\pi_0\left(\gamma_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta\frac{\partial^2}{\partial\theta\,\partial\theta'}U_t\left(\gamma_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\\
-\frac{\sum_{\tilde{\gamma}_t}\frac{\partial^2}{\partial\theta\,\partial\theta'}\exp\left[\pi_0\left(\tilde{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\tilde{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]}{\sum_{\breve{\gamma}_t}\exp\left[\pi_0\left(\breve{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\breve{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]}\\
+\frac{\sum_{\tilde{\gamma}_t}\left\{\left[\frac{\partial}{\partial\theta}\exp\left[\pi_0\left(\tilde{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\tilde{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]\right]\left[\sum_{\hat{\gamma}_t}\frac{\partial}{\partial\theta'}\exp\left[\pi_0\left(\hat{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\gamma_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]\right]\right\}}{\left\{\sum_{\breve{\gamma}_t}\exp\left[\pi_0\left(\breve{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\breve{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]\right\}^2}
\end{array}\right\}\\
&= \begin{pmatrix} I_{1,1} & \cdots & I_{1,12}\\ \vdots & \ddots & \vdots\\ I_{12,1} & \cdots & I_{12,12}\end{pmatrix}\\
\textit{where } I_{i,j} &= \sum_{t=0}^{T}\left\{\begin{array}{l}
\beta\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}U_t\left(\gamma_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\\
-\frac{\sum_{\tilde{\gamma}_t}\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}\exp\left[\pi_0\left(\tilde{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\tilde{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]}{\sum_{\breve{\gamma}_t}\exp\left[\pi_0\left(\breve{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\breve{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]}\\
+\frac{\sum_{\tilde{\gamma}_t}\left\{\left[\frac{\partial}{\partial\theta_i}\exp\left[\pi_0\left(\tilde{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\tilde{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]\right]\left[\sum_{\hat{\gamma}_t}\frac{\partial}{\partial\theta_j}\exp\left[\pi_0\left(\hat{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\hat{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]\right]\right\}}{\left\{\sum_{\breve{\gamma}_t}\exp\left[\pi_0\left(\breve{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)+\beta U_t\left(\breve{\gamma}_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)\right]\right\}^2}
\end{array}\right\}\\
& i=1,2,\ldots,12,\ j=1,2,\ldots,12.
\end{aligned}
\tag{7}
$$

Since

$$
\begin{aligned}
\pi_0\left(\gamma_{it},\vec{x}_{it},\vec{z}_{it};\boldsymbol{\theta}\right) = &-cost\_ins(c_{it})-cost\_adu(a_{it})-cost\_lar(b_{it})\\
&-\theta_1\times l\_same_{it}\times c\_last\_dummy_{it}\\
&-\left[c_{it}=0\right]\times\left\{\begin{array}{l}
\theta_2\times int\_mon\_adu\_discrete_{it}+\\
\theta_3\times int\_mon\_lar\_discrete_{it}+\\
\left[last_{it}\neq -99\right]\times\begin{pmatrix}\theta_4\times int\_ratio\_under_{it}+\\ \theta_5\times int\_ratio\_over_{it}\end{pmatrix}+\\
\left[last_{it}=-99\right]\times\theta_5\times T
\end{array}\right\}\\
&-\left[c_{it}=0\right]\times\left\{\begin{array}{l}
\theta_6\times\left[fly\_m\_disc\_expected_{it}=1\right]+\\
\theta_7\times\left[fly\_m\_disc\_expected_{it}=2\right]+\\
\theta_8\times\left[fly\_f\_disc\_expected_{it}=1\right]+\\
\theta_9\times\left[fly\_f\_disc\_expected_{it}=2\right]+\\
\theta_{10}\times\left[lar\_disc\_expected_{it}=1\right]+\\
\theta_{11}\times\eta_i^{(1)}+\\
\theta_{12}\times\eta_i^{(2)}
\end{array}\right\}\\
&+\varepsilon_{it}\left(\gamma_{it}\right)
\end{aligned}
$$

We have

$$\frac{\partial \pi_0\left(\gamma_{it}, \vec{x}_{it}, \vec{z}_{it}; \boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}} = \begin{pmatrix} -l\_same_{it} \times c\_last\_dummy_{it} \\ -\left[c_{it}=0\right] \times int\_mon\_adu\_discrete_{it} \\ -\left[c_{it}=0\right] \times int\_mon\_lar\_discrete_{it} \\ -\left[c_{it}=0\right] \times \left[last_{it} \neq -99\right] \times int\_ratio\_under_{it} \\ -\left[c_{it}=0\right] \times \left[last_{it} \neq -99\right] \times int\_ratio\_over_{it} \\ -\left[c_{it}=0\right] \times \left[last_{it} = -99\right] \times T \\ -\left[c_{it}=0\right] \times \left[fly\_m\_disc\_expected_{it}=1\right] \\ -\left[c_{it}=0\right] \times \left[fly\_m\_disc\_expected_{it}=2\right] \\ -\left[c_{it}=0\right] \times \left[fly\_f\_disc\_expected_{it}=1\right] \\ -\left[c_{it}=0\right] \times \left[fly\_f\_disc\_expected_{it}=2\right] \\ -\left[c_{it}=0\right] \times \left[lar\_disc\_expected_{it}=1\right] \\ -\left[c_{it}=0\right] \times \eta_i^{(1)} \\ -\left[c_{it}=0\right] \times \eta_i^{(2)} \end{pmatrix} \tag{8}$$

And any derivative taken with respect to $\pi_0(\bullet)$ greater than the first order will be 0.

We have

$$\begin{aligned} &\frac{\partial}{\partial \theta_i} \exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right] \\ &= \exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\left[\frac{\partial}{\partial \theta_i}\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta \frac{\partial}{\partial \theta_i} U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right] \end{aligned} \tag{9}$$

and

$$\begin{aligned} &\frac{\partial^2}{\partial \theta_i\, \partial \theta_j} \exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right] \\ &= \frac{\partial}{\partial \theta_j}\left\{\exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\frac{\partial}{\partial \theta_i}\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\right\} \\ &= \exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\frac{\partial}{\partial \theta_i}\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\frac{\partial}{\partial \theta_j}\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right] + \\ &\quad \exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\frac{\partial^2}{\partial \theta_i\, \partial \theta_j}\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right] \\ &= \exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\frac{\partial}{\partial \theta_i}\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\frac{\partial}{\partial \theta_j}\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right] + \\ &\quad \exp\left[\pi_0\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right]\frac{\partial^2}{\partial \theta_i\, \partial \theta_j}\left[\beta U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)\right] \end{aligned} \tag{10}$$

Let $U_{t,m}\left(\gamma_m, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) = \pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \mid \vec{x}_{t+1}\right]$.

For t = T,

$$U_T\left(\gamma_T, \vec{x}_T, \vec{z}_T; \boldsymbol{\theta}\right) = 0$$

For t = T-1,

$$\begin{aligned}
U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) &= E\left[\log\left\{\sum_{m=1}^{num_action} \exp\left[\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \mid \vec{x}_{t+1}\right]\right]\right\} \mid \gamma_t, \vec{x}_t, \vec{z}_t\right] \\
&= \mathrm{E}\left[\log\left[\sum_{m=1}^{num_action} \exp\left(\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right)\right)\right] \mid \gamma_t, \vec{x}_t, \vec{z}_t\right] \\
\frac{\partial}{\partial\theta_i} U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) &= E_{\vec{x}_{t+1}}\left\{\frac{\partial}{\partial\theta_i} \log\left[\sum_{m=1}^{num_action} \exp\left[\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right)\right]\right]\right\} \\
&= E_{\vec{x}_{t+1}}\left\{\frac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right)\right] \frac{\partial}{\partial\theta_i} \pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right)\right\}}{\sum_{n=1}^{num_action} \exp\left[\pi_0\left(\gamma_n, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right)\right]}\right\} \qquad (11)
\end{aligned}$$

For t < T-1,

$$\begin{aligned}
U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) &= E_{\vec{x}_{t+1}}\left\{\log\left\{\sum_{m=1}^{num_action} \exp\left[\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \middle| \vec{x}_t\right]\right]\right\}\right\} \\
\frac{\partial}{\partial\theta_i} U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) &= E_{\vec{x}_{t+1}}\left\{\frac{\partial}{\partial\theta_i} \log\left\{\sum_{m=1}^{num_action} \exp\left[\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \middle| \vec{x}_t\right]\right]\right\}\right\} \\
&= E_{\vec{x}_{t+1}}\left\{\frac{\sum_{m=1}^{num_action} \frac{\partial}{\partial\theta_i} \exp\left[\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \mid \vec{x}_t\right]\right]}{\sum_{n=1}^{num_action} \exp\left[\pi_0\left(\gamma_n, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \middle| \vec{x}_t\right]\right]}\right\} \\
&= E_{\vec{x}_{t+1}}\left\{\frac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \mid \vec{x}_t\right]\right]\left[\frac{\partial}{\partial\theta_i} \pi_0\left(\gamma_m, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[\frac{\partial}{\partial\theta_i} U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \mid \vec{x}_t\right]\right]\right\}}{\sum_{n=1}^{num_action} \exp\left[\pi_0\left(\gamma_n, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \middle| \vec{x}_t\right]\right]}\right\} \qquad (12) \\
&= E_{\vec{x}_{t+1}}\left\{\frac{\sum_{m=1}^{num_action}\left\{\exp\left(U_{t,m}\right)\left[\frac{\partial}{\partial\theta_i} \pi_0\left(\gamma_m, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) + \beta E_{\vec{x}_{t+1}}\left[\frac{\partial}{\partial\theta_i} U_{t+1}\left(\gamma_{t+1}, \vec{x}_{t+1}, \vec{z}_{t+1}; \boldsymbol{\theta}\right) \mid \vec{x}_t\right]\right]\right\}}{\sum_{n=1}^{num_action} \exp\left(U_{t,n}\right)}\right\}
\end{aligned}$$

So we can get

$$\frac{\partial}{\partial\theta_i} U_t\left(\gamma_t, \vec{x}_t, \vec{z}_t; \boldsymbol{\theta}\right)$$

by backward iteration, and then plug it into the previous functions.

Similarly, we can get

$$\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}U_t\left(\gamma_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)$$

by backward iteration, and then plug it into the previous functions.

For t = T-1,

$$\begin{aligned}
\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}U_t\left(\gamma_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right) &= E_{\vec{x}_{t+1}}\left\{\frac{\partial}{\partial\theta_j}\frac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}}{\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]}\right\} \\
&= E_{\vec{x}_{t+1}}\left\{\begin{array}{l}
\dfrac{\sum_{m=1}^{num_action}\frac{\partial}{\partial\theta_j}\left\{\exp\left[\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}}{\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]} \\
-\dfrac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}\frac{\partial}{\partial\theta_j}\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]}{\left\{\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\right\}^2}
\end{array}\right\} \\
&= E_{\vec{x}_{t+1}}\left\{\begin{array}{l}
\dfrac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_j}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}}{\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]} \\
+\dfrac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}}{\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]} \\
-\dfrac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}\sum_{n=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_j}\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}}{\left\{\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\right\}^2}
\end{array}\right\} \\
&= E_{\vec{x}_{t+1}}\left\{\frac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_j}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}}{\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]}\right\} \\
&\quad -E_{\vec{x}_{t+1}}\left\{\frac{\sum_{m=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}\sum_{n=1}^{num_action}\left\{\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\frac{\partial}{\partial\theta_j}\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right\}}{\left\{\sum_{n=1}^{num_action}\exp\left[\pi_0\left(\gamma_n,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)\right]\right\}^2}\right\}
\end{aligned} \tag{13}$$

For t < T-1,

$$\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}U_t\left(\gamma_t,\vec{x}_t,\vec{z}_t;\boldsymbol{\theta}\right)=E_{\vec{x}_{t+1}}\left\{\frac{\partial}{\partial\theta_j}\frac{\sum_{m=1}^{num\_action}\left\{\exp\left(U_{t,m}\right)\left[\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)+\beta E_{\vec{x}_{t+1}}\left[\frac{\partial}{\partial\theta_i}U_{t+1}\left(\gamma_{t+1},\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)|\,\vec{x}_t\right]\right]\right\}}{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)}\right\}$$

$$=E_{\vec{x}_{t+1}}\left\{\begin{array}{l}\frac{\sum_{m=1}^{num\_action}\frac{\partial}{\partial\theta_j}\left\{\exp\left(U_{t,m}\right)\left[\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)+\beta E_{\vec{x}_{t+1}}\left[\frac{\partial}{\partial\theta_i}U_{t+1}\left(\gamma_{t+1},\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)|\,\vec{x}_t\right]\right]\right\}}{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)}\\ -\frac{\sum_{n=1}^{num\_action}\frac{\partial}{\partial\theta_j}\exp\left(U_{t,n}\right)\sum_{m=1}^{num\_action}\left\{\exp\left(U_{t,m}\right)\left[\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)+\beta E_{\vec{x}_{t+1}}\left[\frac{\partial}{\partial\theta_i}U_{t+1}\left(\gamma_{t+1},\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)|\,\vec{x}_t\right]\right]\right\}}{\left[\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)\right]^2}\end{array}\right\}$$

$$=E_{\vec{x}_{t+1}}\left\{\begin{array}{l}\frac{\sum_{m=1}^{num\_action}\frac{\partial}{\partial\theta_j}\left\{\exp\left(U_{t,m}\right)\left[\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)+\beta E_{\vec{x}_{t+1}}\left[\frac{\partial}{\partial\theta_i}U_{t+1}\left(\gamma_{t+1},\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)|\,\vec{x}_t\right]\right]\right\}}{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)}\\ -\frac{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)\frac{\partial}{\partial\theta_j}\left(U_{t,n}\right)\sum_{m=1}^{num\_action}\exp\left(U_{t,m}\right)\frac{\partial}{\partial\theta_i}\left(U_{t,m}\right)}{\left[\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)\right]^2}\end{array}\right\}$$

$$=E_{\vec{x}_{t+1}}\left\{\begin{array}{l}\frac{\sum_{m=1}^{num\_action}\frac{\partial}{\partial\theta_j}\exp\left(U_{t,m}\right)\left[\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)+\beta E_{\vec{x}_{t+1}}\left[\frac{\partial}{\partial\theta_i}U_{t+1}\left(\gamma_{t+1},\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)|\,\vec{x}_t\right]\right]}{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)}\\ +\frac{\sum_{m=1}^{num\_action}\exp\left(U_{t,m}\right)\left[\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)+\beta E_{\vec{x}_{t+1}}\left[\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}U_{t+1}\left(\gamma_{t+1},\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)|\,\vec{x}_t\right]\right]}{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)}\\ -\frac{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)\frac{\partial}{\partial\theta_j}\left(U_{t,n}\right)\sum_{m=1}^{num\_action}\exp\left(U_{t,m}\right)\frac{\partial}{\partial\theta_i}\left(U_{t,m}\right)}{\left[\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)\right]^2}\end{array}\right\} \tag{14}$$

$$=E_{\vec{x}_{t+1}}\left\{\begin{array}{l}\frac{\sum_{m=1}^{num\_action}\exp\left(U_{t,m}\right)\frac{\partial}{\partial\theta_j}\left(U_{t,m}\right)\frac{\partial}{\partial\theta_i}\left(U_{t,m}\right)}{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)}\\ +\frac{\sum_{m=1}^{num\_action}\beta\exp\left(U_{t,m}\right)E_{\vec{x}_{t+1}}\left[\frac{\partial^2}{\partial\theta_i\,\partial\theta_j}U_{t+1}\left(\gamma_{t+1},\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)|\,\vec{x}_t\right]}{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)}\\ -\frac{\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)\frac{\partial}{\partial\theta_j}\left(U_{t,n}\right)\sum_{m=1}^{num\_action}\exp\left(U_{t,m}\right)\frac{\partial}{\partial\theta_i}\left(U_{t,m}\right)}{\left[\sum_{n=1}^{num\_action}\exp\left(U_{t,n}\right)\right]^2}\end{array}\right\}$$

where $\frac{\partial}{\partial\theta_i}\left(U_{t,m}\right)=\frac{\partial}{\partial\theta_i}\pi_0\left(\gamma_m,\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)+\beta E_{\vec{x}_{t+1}}\left[\frac{\partial}{\partial\theta_i}U_{t+1}\left(\gamma_{t+1},\vec{x}_{t+1},\vec{z}_{t+1};\boldsymbol{\theta}\right)|\,\vec{x}_t\right]$

Hence, plug (8)-(14) into (7), and we will get the Hessian.